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L. W. Hu

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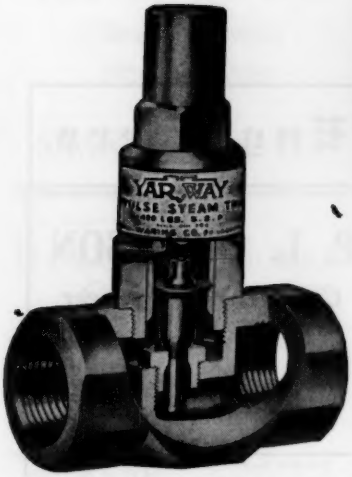

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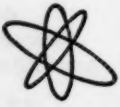

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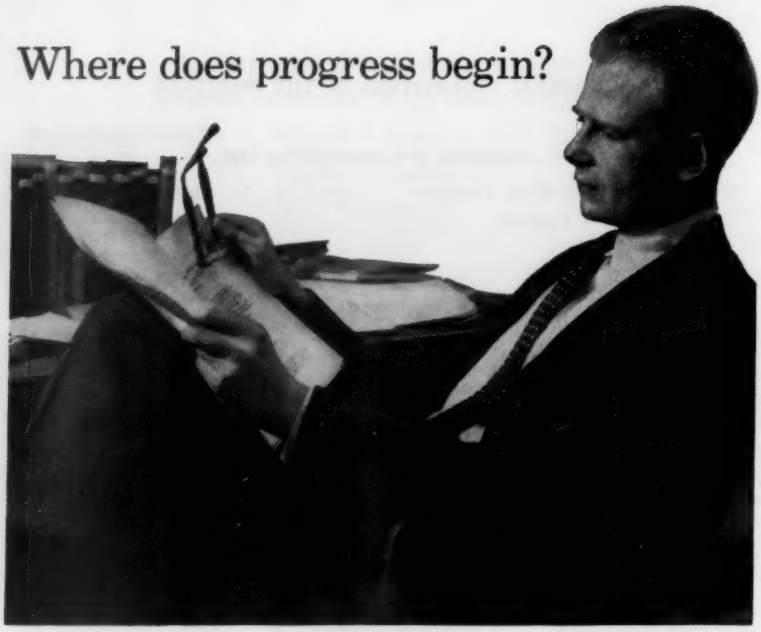

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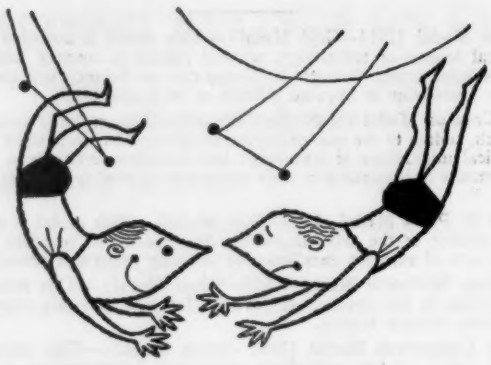

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A SURVEY OF THE USE OF NON-EUCLIDEAN GEOMETRY IN ELECTRICAL ENGINEERING *

BY

E. FOLKE BOLINDER [1]

ABSTRACT

An attempt is made to give a brief resumé of some of the non-Euclidean geometry models and some of their applications in electrical engineering. The survey starts with a discussion of different two-dimensional and three-dimensional non-Euclidean geometry models, both of the hyperbolic and elliptic types. The interconnections of the two-dimensional models are studied geometrically and analytically by means of a simple impedance transformation example. The treatment is then formally extended to three-dimensional models. Finally, some applications of non-Euclidean geometry models in electrical engineering are discussed. An appendix gives a historical note on the evolution of non-Euclidean geometry.

INTRODUCTION

Non-Euclidean geometry has been used in electrical engineering for about ten years. Gradually, engineers and physicists are becoming more interested in using this powerful and convenient tool. Therefore, it seems worth while to try to make a survey of several different non-Euclidean geometry models, their interconnections, and their applications in electrical engineering. Because, of necessity, the survey has to be kept within certain limits, some parts of the presentation are rather condensed. The author has tried, however, to offset this limitation by making frequent reference to important original and expository works. Also, a short historical note on the evolution of non-Euclidean geometry is appended.

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Non-Euclidean geometry is, to use Klein's terminology, divided into hyperbolic geometry, or Gauss-Bolyai-Lobachevsky geometry, and elliptic geometry, or Riemann geometry. Hyperbolic geometry is characterized by the properties that through a point outside a straight line two parallel lines can be drawn, that the sum of the angles of a triangle is less than π, and that this geometry is valid on a surface of constant negative curvature. Elliptic geometry is characterized by the properties that through a point outside a straight line no parallel line can be drawn, that the sum of the angles of a triangle is greater than π, and that this geometry is valid on a surface of constant positive curvature. The limiting case between the two geometrics, parabolic geometry, is conventional Euclidean geometry.

A discussion of the non-Euclidean geometry models follows.

HYPERBOLIC GEOMETRY MODELS

Two-Dimensional Models

1. The pseudosphere. The pseudosphere, which originates if a tractrix is rotated around its asymptote (see Fig. 1) is the simplest surface with constant negative curvature. This important adjunct was combined with hyperbolic geometry by Beltrami (1)[2] in 1868. The pseudosphere has recently been thoroughly discussed by Schilling (2, 3), who also transforms the surface into the complex plane. The pseudosphere has a singular line. It has been shown by Hilbert (4) that in Euclidean space there is no analytic surface of constant negative curvature that does not have a singular line.

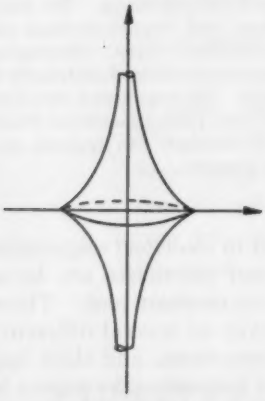

FIG. 1. Tractrix.

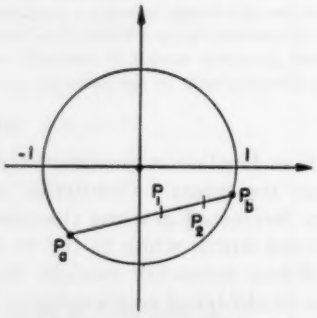

FIG. 2. Two-dimensional Cayley-Klein model.

2. The hyperboloid. Another surface of constant negative curvature which is imbedded in three-dimensional space is the hyperboloid, which

[2] The boldface numbers in parentheses refer to the references appended to this paper.

has been discussed by Schilling (3) and M. Riesz (5). Besides studying the hyperbolic geometry model and the Cayley-Klein and the Poincaré models, Riesz devotes a considerable part of his presentation to a thorough investigation of the properties and movements of the three-dimensional space with Lorentz metric, in which space the hyperboloid is imbedded. For further information on the hyperboloid, see the references in the work of M. Riesz (5).

3. The Cayley-Klein diagram. This model was introduced by Beltrami (1) in 1868. However, it did not come into practical use until 1871, when Klein (6) introduced it as a projective model with a conic as the absolute or fundamental curve. Klein based his work, to a large extent, on the works of Cayley (7); therefore the model is usually called the "Cayley-Klein diagram." An example of this model is shown in Fig. 2. The unit circle in the figure corresponds to the absolute curve (infinity). A straight line is a chord. The hyperbolic distance between two points P_1 and P_2 is defined as half of the logarithm of the cross ratio between

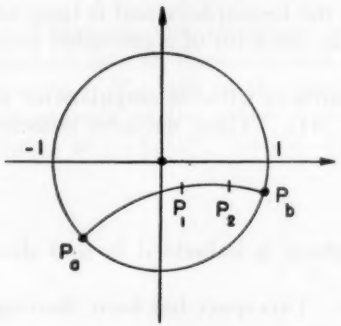

FIG. 3. Two-dimensional Poincaré model.

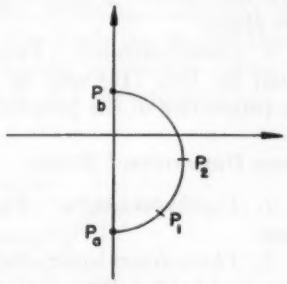

FIG. 4. Two-dimensional Poincaré model.

P_1, P_2, and the two points, P_a and P_b, cut out of the absolute curve by a straight line through P_1 and P_2.

4. The Poincaré model. Although it was known to Beltrami (1) this model has been given Poincaré's name because he used it extensively in his investigations on automorphic functions (8). Examples of this model are shown in Figs. 3 and 4. In Fig. 3 the unit circle is the absolute curve (infinity), while in Fig. 4 the absolute curve is a straight line. In both figures a straight line is represented by an arc of a circle which cuts the absolute curve orthogonally.

Three-Dimensional Models

1. The hyperhyperboloid. The hyperhyperboloid is imbedded in four-dimensional space; thus it is of little interest from the engineer's point of view. However, in many cases the calculations and constructions can be made in three dimensions and formally extended to four dimensions (5). This procedure will be further discussed.

2. The Cayley-Klein model. This model is a direct generalization of the Cayley-Klein diagram to three dimensions. It has been thoroughly studied by Schilling (9), who, for simplicity, uses the (Riemann) unit sphere as an absolute surface.

3. The Poincaré model. This model is a direct generalization of the two-dimensional Poincaré model to three dimensions.

ELLIPTIC GEOMETRY MODELS

Two-Dimensional Models

1. The sphere. The sphere is the simplest surface of constant positive curvature. However, an assumption that two points on the diameter of the sphere together form a single "point" has to be made in order for the sphere (hemisphere) to constitute an ideal model of two-dimensional elliptic geometry. A similar assumption, incidentally, has to be made in connection with the hyperboloid discussed above.

2. The projective plane. A central projection of the hemisphere on a plane parallel to the plane that limits the hemisphere and is tangent to it yields a model of elliptic geometry in the form of a one-sided projective plane.

3. Closed surfaces. Two closed surfaces without singularities were found by Boy (10) and by Schilling (11). These surfaces possess all the properties of the projective plane.

Three-Dimensional Models

1. The hypersphere. The hypersphere is imbedded in four dimensions.

2. Three-dimensional elliptic space. This space has been thoroughly discussed by Schilling (12) and others.

3. Two Riemann spheres. An interesting model of three-dimensional elliptic space consists of two Riemann spheres imbedded in Euclidean space. The natural analytic tool to be used is the theory of quaternions. See the works of Study (13) and others (14).

M. Riesz (5) makes the interesting observation that parabolic geometry is valid on a paraboloid. Thus elliptic geometry is valid on the sphere, parabolic geometry on the paraboloid, and hyperbolic geometry on the hyperboloid.

For additional expository references concerning non-Euclidean geometry models, see references (15–19).

INTERCONNECTIONS OF THE NON-EUCLIDEAN GEOMETRY MODELS

Two-Dimensional Models

1. Geometric treatment. In order to study the interconnections of some of the two-dimensional non-Euclidean geometry models, let us

select a simple example.

$$Z' = k^2 Z = e^{2\psi} Z, \quad k = 2. \tag{1}$$

If the Z's in (1) represent complex impedances, the transformation is performed by an ideal transformer. If we use the well-known transformation,

$$\Gamma' = \frac{Z' - 1}{Z' + 1}, \quad \Gamma = \frac{Z - 1}{Z + 1}, \tag{2}$$

where Γ' and Γ are complex reflection coefficients, (1) transforms into

$$\Gamma' = \frac{\Gamma \cosh \psi + \sinh \psi}{\Gamma \sinh \psi + \cosh \psi} = \frac{\frac{5}{4}\Gamma + \frac{3}{4}}{\frac{3}{4}\Gamma + \frac{5}{4}}. \tag{3}$$

If the complex-impedance plane, $Z = R + jX$, is stereographically mapped on the Riemann unit sphere, the reflection-coefficient plane, $\Gamma = \Gamma_x + j\Gamma_y$, falls in the yz-plane. It can easily be shown that (2) corresponds on the sphere to a rotation of the projection center from (0,0,1) to (−1,0,0). In Fig. 5 the point $Z = 1$ corresponds to $\Gamma = 0$,

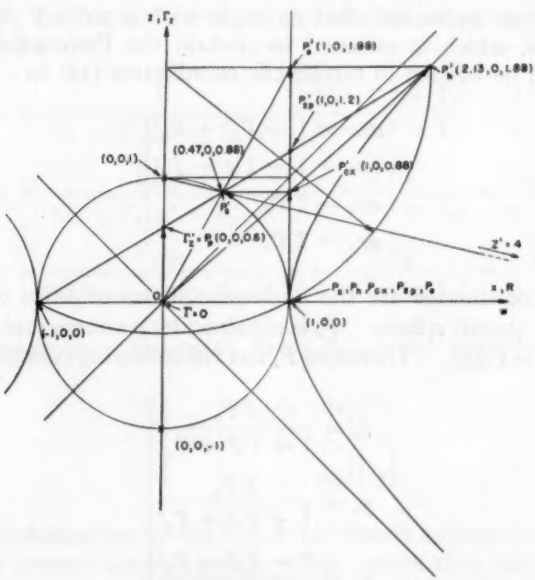

FIG. 5. Connections between different non-Euclidean geometry models.

and $Z' = 4$, obtained from (1), corresponds to $\Gamma_z' = 0.6$. The ideal transformer performs a stretching (hyperbolic transformation) of the surface of the sphere directed from the fixed point $(0,0,-1)$ toward the fixed point (0,0,1,), so that (1,0,0) is transformed into (0.47, 0, 0.833).

We now consider a unit hyperboloid with the x-axis used as the rotation axis. Thus, in Fig. 5 we obtain a unit hyperbola. M. Riesz (5), has shown that if in Fig. 5 the point $\Gamma_z' = P_p$, which may be considered to lie in a two-dimensional Poincaré model in the yz-plane, with the unit circle as the absolute curve, is stereographically mapped on the hyperboloid from $(-1,0,0)$ so that the point P_h' is obtained, then a central line $\overline{OP}_h'$ cuts the vertical plane parallel to the yz-plane through the point (1,0,0) in a point P_{CK}', which may be considered a point of a two-dimensional Cayley-Klein model. The Cayley-Klein diagram may also be obtained by an orthographic projection of the sphere on this vertical plane through (1,0,0). An orthographic projection of P_h' on the same plane yields a point P_e', which may also be obtained by a central projection of the point P_e' on the surface of the sphere. Therefore P_e' lies in an elliptic projective plane. Thus the hyperbolic Cayley-Klein diagram (point P_{CK}'), the ordinary parabolic (Euclidean) plane (point P_{2p}'), and the elliptic projective plane (point P_e') are represented in the same vertical plane. By varying Z' (Γ_z') we immediately obtain a geometric picture of how the points P_e', P_{CK}', P_{2p}', P_e', and P_h' vary. The example selected, $Z = 1 \rightarrow Z' = 4$, is indicated by arrows in Fig. 5.

2. Analytic treatment. Let us begin with a point $\Gamma = \Gamma_z + j\Gamma_y$ in the Γ-plane, which is assumed to contain the Poincaré model. The point Γ can be written in tetracyclic coordinates (14) as

$$\left.\begin{aligned} \sigma x_4 &= 1 + \Gamma_z^2 + \Gamma_y^2 \\ \sigma x_3 &= 1 - \Gamma_z^2 - \Gamma_y^2 \\ \sigma x_2 &= 2\,\Gamma_y \\ \sigma x_1 &= 2\,\Gamma_z \end{aligned}\right\}. \tag{4}$$

These coordinates are the homogeneous coordinates of a point P_e on the unit (hemi) sphere. P_e is obtained by a stereographic projection of Γ from $(-1,0,0)$. Therefore P_e has the following coordinates:

$$\left.\begin{aligned} z_e &= \frac{2\,\Gamma_z}{1 + \Gamma_z^2 + \Gamma_y^2} \\ y_e &= \frac{2\,\Gamma_y}{1 + \Gamma_z^2 + \Gamma_y^2} \\ x_e &= \frac{1 - \Gamma_z^2 - \Gamma_y^2}{1 + \Gamma_z^2 + \Gamma_y^2} \end{aligned}\right\}. \tag{5}$$

Although a point in Fig. 5 is denoted (x,y,z) the order is reversed in the text of the analytic treatment. This is done purposely to facilitate a generalization to higher dimensions which will be carried out later. Thus the equation for the hyperboloid is also written $w^2 - z^2 - y^2 = 1$. (See ref. 5.) The point P_{CK} in the Cayley-Klein diagram has the coordinates

$$\begin{aligned} \Gamma_{zCK} &= \frac{2\,\Gamma_z}{1 + \Gamma_z^2 + \Gamma_y^2} = z_s \\ \Gamma_{yCK} &= \frac{2\,\Gamma_y}{1 + \Gamma_z^2 + \Gamma_y^2} = y_s, \end{aligned} \tag{6}$$

which can be expressed in homogeneous coordinates as

$$\rho \begin{bmatrix} w \\ z \\ y \end{bmatrix} = \begin{bmatrix} \dfrac{1}{\sqrt{1-\Gamma_{zCK}^2-\Gamma_{yCK}^2}} \\ \dfrac{\Gamma_{zCK}}{\sqrt{1-\Gamma_{zCK}^2-\Gamma_{yCK}^2}} \\ \dfrac{\Gamma_{yCK}}{\sqrt{1-\Gamma_{zCK}^2-\Gamma_{yCK}^2}} \end{bmatrix} = \rho \begin{bmatrix} \dfrac{1+\Gamma_z^2+\Gamma_y^2}{1-\Gamma_z^2-\Gamma_y^2} \\ \dfrac{2\,\Gamma_z}{1-\Gamma_z^2-\Gamma_y^2} \\ \dfrac{2\,\Gamma_y}{1-\Gamma_z^2-\Gamma_y^2} \end{bmatrix} = \rho \begin{bmatrix} \dfrac{1}{x_s} \\ \dfrac{z_s}{x_s} \\ \dfrac{y_s}{x_s} \end{bmatrix}. \tag{7}$$

The point P_h on the surface of the hyperboloid $w^2 - z^2 - y^2 = 1$ has the coordinates

$$\left.\begin{aligned} w &= \frac{1}{\sqrt{1 - \Gamma_{zCK}^2 - \Gamma_{yCK}^2}} = \frac{1 + \Gamma_z^2 + \Gamma_y^2}{1 - \Gamma_z^2 - \Gamma_y^2} = \frac{1}{x_s} \\ z &= \frac{\Gamma_{zCK}}{\sqrt{1 - \Gamma_{zCK}^2 - \Gamma_{yCK}^2}} = \frac{2\,\Gamma_z}{1 - \Gamma_z^2 - \Gamma_y^2} = \frac{z_s}{x_s} \\ y &= \frac{\Gamma_{yCK}}{\sqrt{1 - \Gamma_{zCK}^2 - \Gamma_{yCK}^2}} = \frac{2\,\Gamma_y}{1 - \Gamma_z^2 - \Gamma_y^2} = \frac{y_s}{x_s} \end{aligned}\right\}. \tag{8}$$

The transformation of the point P_h to P_h' on the hyperboloid can be expressed as

$$\begin{bmatrix} w' \\ z' \\ y' \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} w \\ z \\ y \end{bmatrix}. \tag{9}$$

Here the transformation by the 3×3 real matrix belongs to the G_+ subgroup of the proper Lorentz group, which means that the elements of the matrix have to satisfy the six following conditions (20):

$$\left.\begin{aligned} a_1^2 - b_1^2 - c_1^2 &= -1 \\ a_2^2 - b_2^2 - c_2^2 &= -1 \\ a_3^2 - b_3^2 - c_3^2 &= 1 \\ a_1a_2 - b_1b_2 - c_1c_2 &= 0 \\ a_2a_3 - b_2b_3 - c_2c_3 &= 0 \\ a_1a_3 - b_1b_3 - c_1c_3 &= 0 \end{aligned}\right\}. \tag{10}$$

The transformed point P_{CK} in the Cayley-Klein diagram is in homogeneous coordinates:

$$\rho \begin{bmatrix} w' \\ z' \\ y' \end{bmatrix}$$

and the point P_s' on the unit sphere is given by

$$\left.\begin{aligned} z_s' &= \frac{z'}{w'} = \frac{z'}{\sqrt{1 + z'^2 + y'^2}} \\ y_s' &= \frac{y'}{w'} = \frac{y'}{\sqrt{1 + z'^2 + y'^2}} \\ x_s' &= \frac{1}{w'} = \frac{1}{\sqrt{1 + z'^2 + y'^2}} \end{aligned}\right\}. \tag{11}$$

P_{CK}' in the Cayley-Klein diagram is now simply

$$\left.\begin{aligned} \Gamma_{zCK}' &= z_s' \\ \Gamma_{yCK}' &= y_s' \end{aligned}\right\}. \tag{12}$$

Finally, the transformed point $\Gamma' = \Gamma_z' + j\Gamma_y'$ in the Γ-plane (Poincaré model) is given by

$$\left.\begin{aligned} \Gamma_z' &= \frac{z_s'}{1+x_s'} = \frac{z'}{1+\sqrt{1+z'^2+y'^2}} = \frac{\Gamma_{zCK}'}{1+\sqrt{1-\Gamma_{zCK}'^2-\Gamma_{yCK}'^2}} \\ \Gamma_y' &= \frac{y_s'}{1+x_s'} = \frac{y'}{1+\sqrt{1+z'^2+y'^2}} = \frac{\Gamma_{yCK}'}{1+\sqrt{1-\Gamma_{zCK}'^2-\Gamma_{yCK}'^2}} \end{aligned}\right\}. \tag{13}$$

The rather complicated procedure for transforming Γ into Γ', which is presented above, has been performed in order to show analytically how some of the two-dimensional non-Euclidean geometry models can be used. In the presentation we have gone from the Γ-plane (Poincaré model) to the hyperboloid by means of the sphere and the Cayley-

Klein model; we might just as well have used the elliptic projective plane instead of the Cayley-Klein model.

Three-Dimensional Models

1. Geometric treatment. In order to show the interconnections of the three-dimensional models, we have to use a four-dimensional space. We shall find, however, that some of the projections performed result in three-dimensional models. These models, which are generalizations of the plane models in the two-dimensional case, can be used for direct geometric constructions.

2. Analytic treatment. Let us begin with a point $Q = (Q_x, Q_y, Q_z)$ in three-dimensional space (Poincaré model). The point Q can be written in pentaspheric coordinates (14) as:

$$\left.\begin{aligned} \sigma x_5 &= 1 + Q_x^2 + Q_y^2 + Q_z^2 \\ \sigma x_4 &= 1 - Q_x^2 - Q_y^2 - Q_z^2 \\ \sigma x_3 &= 2Q_x \\ \sigma x_2 &= 2Q_y \\ \sigma x_1 &= 2Q_z \end{aligned}\right\}. \tag{14}$$

These coordinates are the homogeneous coordinates of a point Q_s on the unit (hemi) hypersphere $z^2 + y^2 + x^2 + v^2 = 1$. Therefore, Q_s has the following coordinates:

$$\left.\begin{aligned} z_s &= \frac{2Q_x}{1 + Q_x^2 + Q_y^2 + Q_z^2} \\ y_s &= \frac{2Q_y}{1 + Q_x^2 + Q_y^2 + Q_z^2} \\ x_s &= \frac{2Q_z}{1 + Q_x^2 + Q_y^2 + Q_z^2} \\ v_s &= \frac{1 - Q_x^2 - Q_y^2 - Q_z^2}{1 + Q_x^2 + Q_y^2 + Q_z^2} \end{aligned}\right\}. \tag{15}$$

The corresponding point Q_{CK} in the Cayley-Klein model is:

$$\left.\begin{aligned} Q_{xCK} &= \frac{2Q_x}{1 + Q_x^2 + Q_y^2 + Q_z^2} = z_s \\ Q_{yCK} &= \frac{2Q_y}{1 + Q_x^2 + Q_y^2 + Q_z^2} = y_s \\ Q_{zCK} &= \frac{2Q_z}{1 + Q_x^2 + Q_y^2 + Q_z^2} = x_s \end{aligned}\right\}. \tag{16}$$

In homogeneous coordinates the point Q_{CK} is given by

$$\rho\begin{pmatrix} w \\ z \\ y \\ x \end{pmatrix} = \rho\begin{pmatrix} \dfrac{1}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} \\ \dfrac{Q_{xCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} \\ \dfrac{Q_{yCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} \\ \dfrac{Q_{zCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} \end{pmatrix} = \rho\begin{pmatrix} \dfrac{1+Q_x^2+Q_y^2+Q_z^2}{1-Q_x^2-Q_y^2-Q_z^2} \\ \dfrac{2Q_x}{1-Q_x^2-Q_y^2-Q_z^2} \\ \dfrac{2Q_y}{1-Q_x^2-Q_y^2-Q_z^2} \\ \dfrac{2Q_z}{1-Q_x^2-Q_y^2-Q_z^2} \end{pmatrix} = \rho\begin{pmatrix} \dfrac{1}{v_0} \\ \dfrac{z_0}{v_0} \\ \dfrac{y_0}{v_0} \\ \dfrac{x_0}{v_0} \end{pmatrix} \tag{17}$$

The point Q_h on the surface of the hyperhyperboloid $w^2 - z^2 - y^2 - x^2 = 1$ is given by

$$\left.\begin{aligned} w &= \frac{1}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} = \frac{1+Q_x^2+Q_y^2+Q_z^2}{1-Q_x^2-Q_y^2-Q_z^2} = \frac{1}{v_0} \\ z &= \frac{Q_{xCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} = \frac{2Q_x}{1-Q_x^2-Q_y^2-Q_z^2} = \frac{z_0}{v_0} \\ y &= \frac{Q_{yCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} = \frac{2Q_y}{1-Q_x^2-Q_y^2-Q_z^2} = \frac{y_0}{v_0} \\ x &= \frac{Q_{zCK}}{\sqrt{1-Q_{xCK}^2-Q_{yCK}^2-Q_{zCK}^2}} = \frac{2Q_z}{1-Q_x^2-Q_y^2-Q_z^2} = \frac{x_0}{v_0} \end{aligned}\right\} \tag{18}$$

The transformation of the point Q_h to Q_h' on the hyperhyperboloid can be expressed as

$$\begin{pmatrix} w' \\ z' \\ y' \\ x' \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{pmatrix}\begin{pmatrix} w \\ z \\ y \\ x \end{pmatrix}. \tag{19}$$

Here the transformation by the 4×4 real matrix belongs to the G_+ subgroup of the proper Lorentz group. This means that the elements of the matrix have to satisfy the ten following conditions (**9**, **20**):

$$\left.\begin{aligned} a_1^2 - b_1^2 - c_1^2 - d_1^2 &= -1 \\ a_2^2 - b_2^2 - c_2^2 - d_2^2 &= -1 \\ a_3^2 - b_3^2 - c_3^2 - d_3^2 &= -1 \\ a_4^2 - b_4^2 - c_4^2 - d_4^2 &= 1 \\ a_1a_2 - b_1b_2 - c_1c_2 - d_1d_2 &= 0 \\ a_2a_3 - b_2b_3 - c_2c_3 - d_2d_3 &= 0 \\ a_3a_4 - b_3b_4 - c_3c_4 - d_3d_4 &= 0 \\ a_1a_3 - b_1b_3 - c_1c_3 - d_1d_3 &= 0 \\ a_2a_4 - b_2b_4 - c_2c_4 - d_2d_4 &= 0 \\ a_1a_4 - b_1b_4 - c_1c_4 - d_1d_4 &= 0 \end{aligned}\right\} \tag{20}$$

The transformed point Q_{CK}' in the Cayley-Klein model is in homogeneous coordinates:

$$\rho \begin{bmatrix} w' \\ z' \\ y' \\ x' \end{bmatrix}$$

and the point Q_s on the hypersphere is given by

$$\left.\begin{aligned} z_s' &= \frac{z'}{w'} = \frac{z'}{\sqrt{1 + z'^2 + y'^2 + x'^2}} \\ y_s' &= \frac{y'}{w'} = \frac{y'}{\sqrt{1 + z'^2 + y'^2 + x'^2}} \\ x_s' &= \frac{x'}{w'} = \frac{x'}{\sqrt{1 + z'^2 + y'^2 + x'^2}} \\ v_s' &= \frac{1}{w'} = \frac{1}{\sqrt{1 + z'^2 + y'^2 + x'^2}} \end{aligned}\right\}. \tag{21}$$

Q_{CK}' in the three-dimensional Cayley-Klein model is now simply

$$\left.\begin{aligned} Q_{zCK}' &= z_s' \\ Q_{yCK}' &= y_s' \\ Q_{xCK}' &= x_s' \end{aligned}\right\}. \tag{22}$$

Finally, the transformed point $Q' = (Q_x', Q_y', Q_z')$ in the original space (Poincaré model) is given by

$$\left.\begin{aligned} Q_x' &= \frac{z_s'}{1 + v_s'} = \frac{z'}{1 + \sqrt{1 + z'^2 + y'^2 + x'^2}} = \frac{Q_{zCK}'}{1 + \sqrt{1 - Q_{zCK}'^2 - Q_{yCK}'^2 - Q_{xCK}'^2}} \\ Q_y' &= \frac{y_s'}{1 + v_s'} = \frac{y'}{1 + \sqrt{1 + z'^2 + y'^2 + x'^2}} = \frac{Q_{yCK}'}{1 + \sqrt{1 - Q_{zCK}'^2 - Q_{yCK}'^2 - Q_{xCK}'^2}} \\ Q_z' &= \frac{x_s'}{1 + v_s'} = \frac{x'}{1 + \sqrt{1 + z'^2 + y'^2 + x'^2}} = \frac{Q_{xCK}'}{1 + \sqrt{1 - Q_{zCK}'^2 - Q_{yCK}'^2 - Q_{xCK}'^2}} \end{aligned}\right\}. \tag{23}$$

In the presentation given above we have gone from Q-space (Poincaré model) to the hyperhyperboloid by means of the hypersphere and the Cayley-Klein model. Instead of the Cayley-Klein model we could just as well have used the three-dimensional elliptic space.

SOME APPLICATIONS OF NON-EUCLIDEAN GEOMETRY MODELS IN ELECTRICAL ENGINEERING

In 1931, König (21) mapped the complex-impedance plane stereographically on a sphere. In order to be able to use a transmission-line equivalent network for a given network, he restricts himself to the study of symmetric networks. He considers an impedance transformation through a lossy, bilateral, symmetric, two-port network as corresponding to a spiral movement of the surface of the sphere. By selecting the radius of the sphere equal to the absolute value of the image impedance of the network, the axis of the spiral movement is made to pass through the center of the sphere. König splits the transformation, which is called a "loxodromic" transformation, into a pure rotation around the axis, corresponding to an elliptic transformation, and a pure stretching along the axis, corresponding to a hyperbolic transformation. The amount of rotation and stretching is given by a theorem on the invariance of a cross ratio. If the sphere is used as the absolute surface of a three-dimensional Cayley-Klein model of non-Euclidean hyperbolic geometry, the transformation can easily be performed by non-Euclidean geometry methods. Apparently, König was not aware of this convenient tool. In the special case which he used, the rotation was easily performed, since the axis of the transformation passes through the center of the sphere, and it merely consists of a rotation through a given Euclidean angle. But the stretching, which consists basically of a translation of a point on the axis through a given hyperbolic distance, was more difficult to perform. König had to use a calibration curve that enabled him to convert a hyperbolic distance into a Euclidean distance. After adding two Euclidean distances, he then had to reconvert. Although he did not make any use of non-Euclidean geometry in his treatment of lossy, two-port networks, König's paper is mentioned here because of his use of the sphere and his splitting of the transformation, which entitle him to be a forerunner of the application of non-Euclidean geometry to electrical engineering.

Two-Dimensional Models

Van Slooten, who published a thesis in 1946 on "Geometric Considerations in Connection with the Theory of Four-Terminal Networks" (22), seems to have been the first to introduce non-Euclidean geometry into electrical engineering. This work is written in Dutch, but it is accompanied by an extensive summary in English with references to the figures in the text. Van Slooten sets the impedance transformations through bilateral, lossless, two-port networks in correspondence with movements in two-dimensional hyperbolic geometry models. One of Van Slooten's research objectives is to find the resultant network of two cascaded lossless networks. This corresponds to finding the resultant

movement to two known movements in the hyperbolic plane. Van Slooten begins with a study of the Poincaré model but does not succeed in finding the resultant movement. He therefore converts the Poincaré model into a Cayley-Klein model. In this model he succeeds in finding the desired construction.

In order to compress the complex-impedance plane into a practical area, Steiner (**23**), too in 1946, mapped the plane on the Riemann sphere and performed an orthographic projection on the *yz*-plane. From Fig. 5 we recognize that the impedance plane is actually transformed into the two-dimensional Cayley-Klein model in the reflection-coefficient plane. Steiner applies the diagram to electrical problems, but nowhere does he mention non-Euclidean geometry.

Not knowing of the work of Van Slooten and Steiner, Deschamps introduced the two-dimensional Cayley-Klein model in 1951 in order to facilitate polarization-ratio transformations (**24**). His diagram was obtained by an orthographic projection of the Poincaré sphere. Later he developed some powerful tools for working with hyperbolic geometry, in that he showed how a non-Euclidean (elliptic) angle can be measured by transforming it to the center of the diagram, and how a hyperbolic distance can be measured by means of an ingenious hyperbolic protractor of his own design (**25**, **26**). Deschamps (**27**) worked a series of examples of impedance and polarization-ratio transformations showing the usefulness of these tools; he also gave a thorough discussion of the Poincaré model. Using two-dimensional hyperbolic geometry models, he showed simple graphical constructions for finding the scattering matrix of a two-port network (**28**). The same problem was solved by Storer, Sheingold, and Stein (**29**) in a paper that was based on an U.R.-S.I. address and some Federal Telecommunications Laboratories reports by Deschamps. However, Storer, Sheingold, and Stein carefully avoided the use of non-Euclidean geometry and based their proofs on inversion methods. With the use of inversion they developed a graphical method for transforming a reflection coefficient through a lossy, bilateral, two-port network. The method consists basically of a geometric interpretation of an analytic formula. Their method was criticized by Deschamps (**30**), who pointed out that the simpler method, using the hyperbolic protractor and the elliptic-angle construction, can be performed inside the unit circle and, therefore, there is no need for an alternative construction such as theirs. Despite this criticism, the inversion method has gained a thorough hearing in a recent textbook (**31**).

In a series of six papers de Buhr (**32**–**37**) made a thorough study of performing impedance transformations through bilateral, two-port networks geometrically by straightedge and compass. In his treatment he frequently makes use of both the two-dimensional Cayley-Klein model and the Poincaré model. Evidently, de Buhr is not familiar with Van

Slooten's work and only partly familiar with Deschamps' (he refers to two of Deschamps' papers in his first paper). Some of de Buhr's constructions are very interesting. As an example, an elegant method for finding the resultant transformer of two cascaded lossless transformers can be mentioned (**36**). In his sixth paper (**37**) de Buhr describes a graphical method for transforming complex impedances through lossless, two-port networks. The writer (**38**) found this method independently by a simple extension of a method of Van Slooten.

Three-Dimensional Models

In his thesis Van Slooten (**22**) states that just as the two-dimensional hyperbolic model is suited for impedance transformations through lossless, two-port networks, the three-dimensional hyperbolic model ought to be applicable for impedance transformations through lossy, two-port networks. He adds, however, that the use of the three-dimensional hyperbolic model seems to be too complicated to be of any use in electrical engineering.

Steiner (**23**) makes a similar statement, saying that, in practice, one cannot work with the Riemann sphere itself.

Deschamps, in a basic paper (**39**), extends the circular transformations on the sphere to the surrounding space E^3. Points of this space are set to correspond to Hermitian forms of a wave vector. In the space E^3 Deschamps introduces a metric which is related to the metric in a four-dimensional Minkowski space. After a brief discussion of the three-dimensional Cayley-Klein model, Deschamps gives a thorough treatment of conventional spherical geometry and of the two-dimensional Cayley-Klein model.

In the course of an investigation on the use of modern (higher) geometry for solving microwave problems the writer recently introduced a graphical method for impedance and polarization-ratio transformations by the linear-fractional transformation. This method (**40, 41**) is called the "isometric circle method" because it utilizes two circles called the "isometric circles." One operation consists of an inversion in one of the isometric circles. But it is not very practical to use this method with circles of large radii. In order to compress the complex-impedance plane, the writer began the study of hyperbolic-geometry models. The two-dimensional Cayley-Klein model turned out to be very suitable for transfer of the isometric circle method in cases of impedance transformations through lossless, two-port networks (**38**). Similarly, the three-dimensional Cayley-Klein model was equally suited to impedance transformations through lossy, two-port networks (**42, 43**). Further study revealed, for example, that the three-dimensional Cayley-Klein model could be constructively used for determining the efficiency of two-port networks (**44**), for creating a general method of cascading two-port

networks by the "Schilling figure" (42, 43), and for creating a general method of analyzing two-port networks from three arbitrary measurements (43, 45). The Schilling figure consists of six straight lines all of which cut the sphere and all of which are consecutively non-Euclidean perpendicular (9).

APPENDIX

HISTORICAL NOTE ON THE EVOLUTION OF NON-EUCLIDEAN GEOMETRY

The evolution of non-Euclidean geometry followed two different lines of development. The oldest started with Euclid's fifth postulate ("parallel postulate") which states:

"That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles."

For 2000 years attempts were made to derive the fifth postulate from the first four postulates of Euclid, but without success. C. F. Gauss (1777–1855) was the first to invent a geometry that substituted for Euclid's fifth postulate another hypothesis but retained all the others. Some of this geometry, which Gauss called "non-Euclidean," was formulated as early as 1792, but he resolved not to publish it during his lifetime. In this geometry the sum of the angles of a triangle is always less than π, and through a point outside a straight line two parallel lines can be drawn. This new geometry was invented independently in 1823 by J. Bolyai (1802–1860) and published in 1832, and by N. I. Lobachevsky (1793–1856), who presented his researches to the Kasan Scientific Society in 1826 but did not secure publication until 1829.

In both the Euclidean and the Gauss-Bolyai-Lobachevsky geometries the assumption is made that the straight line is infinite in extent. B. Riemann (1826–1866) recognized another possibility. In his famous trial lecture, read before the philosophical faculty of University of Göttingen on June 10, 1854, he described a geometry in which a straight line is finite and closed. In this geometry the sum of the angles of a triangle is always greater than π, and through a point outside a straight line no parallel line can be drawn. Riemann's lecture (46) was not published until 1868, after his death. It has been republished and commented upon by Weyl (47), and it has been translated into English by Clifford (46).

The second line of development of non-Euclidean geometry grew out of the "projective geometry" of Desargues, and others which was brought to fruition in 1822 by J. V. Poncelet (1788–1867). He showed that in Euclidean geometry the formula for distance is closely related to a degenerate quadratic form that corresponds to the imaginary circular points. This "fundamental" or "absolute" expression serves as a basis for all Euclidean measure. In 1859 (7), in the sixth in a series of papers, entitled "Memoirs upon Quantics," A. Cayley (1821–1895) selected as a basis, instead of the imaginary circular points, an arbitrary quadratic form, and defined, in correspondence with it, distance and angle in such a way that the Euclidean quantities were obtained when the quadratic form degenerated into the imaginary circular points. As Cayley was not aware of the non-Euclidean geometry works mentioned above, it became the important contribution of F. Klein (1849–1925) to show that non-Euclidean geometry could be treated in a simple way by using projective geometry. Klein (6) published his results in 1871. In this paper he coined the terms "hyperbolic geometry" for Gauss-Bolyai-Lobachevsky geometry, "parabolic geometry" for Euclidean geometry, and "elliptic geometry" (or "spherical geometry") for Riemann geometry.

For excellent surveys of non-Euclidean geometry, see, for example, (15), (16), (19), and (48).

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MODIFIED TRESCA'S YIELD CONDITION AND ASSOCIATED FLOW RULES FOR ANISOTROPIC MATERIALS AND APPLICATIONS*

BY

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SUMMARY

A modified Tresca's yield condition with associated flow rules for anisotropic materials was suggested. By using this approach, it was shown that problems of which the principal axes of anisotropy coincide with the principal axes of stress, can be solved simply. Examples having known solutions for isotropic cases were solved. The results demonstrated that great error may be resulted by postulating the plastic flow of a strained body of anisotropic material on the basis of its behavior if it were isotropic.

1. INTRODUCTION

In experimental studies of the plastic flow of metals, it has been found that for some materials, the yield stress and the plastic stress-strain relations vary with direction. For example, in a cold-rolled metal, the yield stress transverse to the rolling direction may differ considerably from the yield stress parallel to the rolling direction. Moreover, for materials such as cast iron, it has been found that the tensile yield stress and the compressive one in the same orientation are not equal in magnitude. These kinds of anisotropy can be described by formulating the anisotropic yield conditions which specify the stress states under which plastic flow will occur (**1**, **2**).[2] However, in the analytical studies of the plastic flow of anisotropic bodies, the mathematical complications due to using the anisotropic yield conditions become almost insurmountable in many cases. Up to date, only a few problems in the plastic flow of anisotropic materials have been solved (**3**). The directional anisotropy has been studied by Hill and other investigators using v. Mises' yield condition and its flow rules. Only in the study of plastic bending has the difference in tensile and compressive yield stresses been considered (**4**).

In this paper, a modified Tresca's yield condition and its associated flow rules for anisotropic materials are suggested by extending the concept of the Tresca's yield condition and its associated flow rules for isotropic solids advanced by Prager (**5**). By using this approach, it is shown that certain groups of problems in plasticity of anisotropic solids

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

can be solved simply. The solutions, otherwise, would be very much involved or almost impossible to obtain. Two problems with known solutions for isotropic cases will be discussed to illustrate the applicability of the modified Tresca's yield condition as well as to show the effects of anisotropy on the plastic flow of metals. Furthermore, these studies will show that it is impossible to postulate the plastic flow of a strained body of anisotropic material on the basis of its behavior if it were isotropic.

2. YIELD CONDITION FOR ANISOTROPIC MATERIALS

For isotropic perfectly plastic materials, the yield condition to specify the stress state under which the plastic flow may occur can be expressed in the form

$$F(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}, \sigma_0) = 0 \tag{1}$$

which can be considered as a surface in a six dimensional space. That is, a point on the surface is defined by six variables σ_x, σ_y, σ_z, τ_{xy}, τ_{yz} and τ_{zx}. However, such a stress state can be transformed into an equivalent stress state defined by three principal stresses σ_I, σ_{II} and σ_{III}. Because the material is isotropic, the orientation of the principal axes of stresses does not enter into the yield condition. Therefore, the yield condition Eq. 1 can be written as

$$F(\sigma_I, \sigma_{II}, \sigma_{III}, \sigma_0) = 0. \tag{2}$$

Geometrically, this yield condition can be represented by a prismatic or cylindrical surface in the stress space. The stress components σ_I, σ_{II} and σ_{III} are considered as the rectangular coordinates of a point on the yield surface. On account of the postulation that the hydrostatic pressure produces no effects on the plastic flow of metals, the yield surface must be generated by lines parallel to the line passing through the origin with the direction cosines $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$.

In the case of anisotropic materials, the orientation of the principal axes of stresses becomes a factor to be considered in the formulation of the yield condition. For simplicity, let us consider anisotropic solids possessing three orthogonal planes of symmetry in anisotropy at every point. The intersections of these planes are called the principal axes of anisotropy (6). No shear deformation will be produced by the normal stresses applied in the directions of the principal axes of anisotropy. For such an anisotropic material, the yield condition can be formulated in the similar manner as Eq. 1:

$$F(\sigma_1, \sigma_2, \sigma_3, \tau_{12}, \tau_{23}, \tau_{31}, \sigma_{10}, \sigma_{20}, \sigma_{30}, \tau_{120}, \tau_{230}, \tau_{310}) = 0, \tag{3}$$

where the subscripts 1, 2, and 3, refer to the principal axes of anisotropy, and the subscript 0 denotes the yield stresses. Because the hydrostatic

pressure is assumed to have no effect on plastic flow, the values of the six yield stresses are not entirely independent of each other. The yield condition Eq. 3 can be expressed in terms of the principal stresses of the stress state $(\sigma_1, \sigma_2, \sigma_3, \tau_{12}, \tau_{23}, \tau_{31})$. However, this does not simplify the expression of the yield condition, because the orientation of the principal axes of stresses should be taken into consideration. If Euler's angles are used to describe the relative position of the principal axes of stress to the principal axes of anisotropy, the yield condition may be written as

$$F(\sigma_I, \sigma_{II}, \sigma_{III}, \psi, \phi, \theta, \sigma_{10}, \sigma_{20}, \sigma_{30}, \tau_{120}, \tau_{230}, \tau_{310}) = 0. \tag{4}$$

Therefore, the plastic flow of an anisotropic solid can be investigated if the orientation of the principal axes of stress to the principal axes of anisotropy during deformation is known. For this reason, the problems in the plastic flow of anisotropic solids are complicated. Up to date, only few special cases have been studied. One of the cases is that in which the principal axes of stresses coincide with the principal axes of anisotropy. Another case is that in which the stress components σ_3, τ_{23} and τ_{31} vanish.

In 1953, Prager pointed out the merits of using Tresca's yield condition and its associated flow rules to study the plastic flow of isotropic materials (**5**). It has been found that such an approach can yield solutions to many problems (**5**, **7**, **8**). Considering the nature of this approach in the light of the above discussion on the anisotropic loading functions, one may see that this method can be extended to solve certain problems in plastic flow of anisotropic materials, provided that the principal axes of anisotropy coincided with the principal axes of stress. For this type of problems, a modified Tresca's yield condition for anisotropic materials can be constructed and its associated flow rules can be specified as in the following section.

3. MODIFIED TRESCA'S YIELD CONDITION AND ASSOCIATED FLOW RULES

Consider an anisotropic solid whose state of anisotropy possesses three orthogonal planes of symmetry. If the principal axes of stress coincide with the principal axes of anisotropy at all points when the solid is plastically deformed, the yield condition is reduced to the form

$$F(\sigma_1, \sigma_2, \sigma_3, \sigma_{10}, \sigma_{20}, \sigma_{30}) = 0. \tag{5}$$

Geometrically, the yield condition Eq. 5 can be represented by a prismatic or cylindrical surface in the stress space. If hydrostatic pressure is assumed to produce no effects on the plastic flow, the yield surface must be generated by the lines parallel to the line passing through the origin with direction cosines $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. Therefore, although

the yield stresses in three principal directions may be different, they are interrelated in such a manner as to fulfil the requirements imposed by the assumption aforementioned.

In two-dimensional cases, the yield condition can be represented by the intersection of the yield surface with one principal plane as shown in Fig. 1. The first subscript indicates the direction, the second subscript designates the yield stress and the superscript distinguishes between tension and compression. Following the same reasoning employed by Prager on the use of singular yield conditions for isotropic

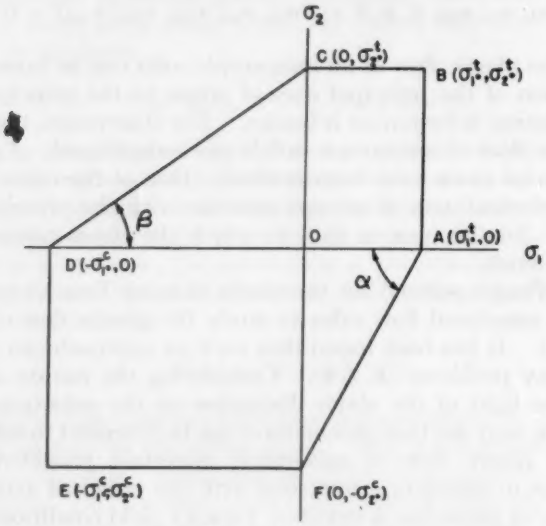

FIG. 1. Modified Tresca's yield condition.

solids, the three direction cosines of the exterior normal of the yield surface at a point are proportional to the three principal plastic strain rates $\dot{\epsilon}_1$, $\dot{\epsilon}_2$ and $\dot{\epsilon}_3$. For a point on the edge formed by two adjacent faces, each of which has a unique normal at this point, principal strain rates $\dot{\epsilon}_1$, $\dot{\epsilon}_2$, $\dot{\epsilon}_3$ are assumed to be proportional to a linear combination of the two sets of direction cosines of the two normals. By considering that no volume change is produced by plastic deformation, the flow rules for the states of stress represented by the specified region in Fig. 1 are given in Table I. For all cases, the coefficient λ is positive and varies

TABLE I.—*Associated Flow Rules.*

Region	Stress State	Flow Rules $e_1:e_2:e_3$
A	$\sigma_1 = \sigma_{10}^t,\ \sigma_2 = 0$	$\sqrt{2}(1-\lambda)\cos\alpha + \lambda$: $-\sqrt{2}(1-\lambda)\sin\alpha$: $\sqrt{2}(1-\lambda)(\sin\alpha - \cos\alpha) - \lambda$
A–B	$\sigma_1 = \sigma_{10}^t,\ 0 \leqslant \sigma_2 \leqslant \sigma_{20}^t$	$1:0:-1$
B	$\sigma_1 = \sigma_{10}^t,\ \sigma_2 = \sigma_{20}^t$	$1-\lambda:\lambda:-1$
B–C	$0 \leqslant \sigma_1 \leqslant \sigma_{10}^t,\ \sigma_2 = \sigma_{20}^t$	$0:1:-1$
C	$\sigma_1 = 0,\ \sigma_2 = \sigma_{20}^t$	$-\sqrt{2}\lambda\sin\beta$: $(1-\lambda) + \sqrt{2}\lambda\cos\beta$: $-(1-\lambda) + \sqrt{2}\lambda(\sin\beta - \cos\beta)$
C–D	$(\sigma_1/\sigma_{10}^c) - (\sigma_2/\sigma_{20}^t) = -1$	$-\sqrt{2}\sin\beta:\sqrt{2}\cos\beta$: $\sqrt{2}(\sin\beta - \cos\beta)$
D	$\sigma_1 = -\sigma_{10}^c,\ \sigma_2 = 0$	$-\sqrt{2}(1-\lambda)\sin\beta - \lambda$: $\sqrt{2}(1-\lambda)\cos\beta$: $\sqrt{2}(1-\lambda)(\sin\beta - \cos\beta) + \lambda$
D–E	$\sigma_1 = -\sigma_{10}^c,\ 0 \geqslant \sigma_2 \geqslant -\sigma_{20}^c$	$-1:0:1$
E	$\sigma_1 = -\sigma_{10}^c,\ \sigma_2 = -\sigma_{20}^c$	$-(1-\lambda):-\lambda:1$
E–F	$0 \geqslant \sigma_1 \geqslant -\sigma_{10}^c,\ \sigma_2 = -\sigma_{20}^c$	$0:-1:1$
F	$\sigma_1 = 0,\ \sigma_2 = -\sigma_{20}^c$	$\sqrt{2}(1-\lambda)\cos$: $-\sqrt{2}(1-\lambda)\sin\alpha - \lambda$: $-\sqrt{2}(1-\lambda)(\sin\alpha - \cos\alpha) + \lambda$
F–A	$(\sigma_1/\sigma_{10}^t) - (\sigma_2/\sigma_{20}^c) = 1$	$\sqrt{2}\cos\alpha:-\sqrt{2}\sin\alpha$: $\sqrt{2}(\sin\alpha - \cos\alpha)$

$\lambda: 0 \leqslant \lambda \leqslant 1$
$\alpha: \tan\alpha = (\sigma_{20}^c/\sigma_{10}^t)$
$\beta: \tan\beta = (\sigma_{20}^t/\sigma_{10}^c)$

from zero to unity. When the yield stresses are of the same value, the flow rules for the anisotropic materials in Table I will reduce to that for isotropic solids (5). It should be pointed out that the yield stress in the third principal direction does not enter into the flow rules given in Table I. However, the magnitudes of yield stresses are not completely independent if the hydrostatic pressure is postulated to produce no effects on the plastic flow of metals.

The effects of anisotropy on the flow rules can be seen in Table I. For instance, it is interesting to notice that the plastic flow in the third principal direction can vanish only for the stress states represented by the points A, C, D, F when the coefficient λ has to assume particular values. On the other hand, for isotropic materials, the plastic flow in

the third principal direction always vanishes for the stress states represented by the segments corresponding to *C-D* and *F-A*. The parameters of anisotropy may not appear in the flow rules in some regimes as shown in Table I, but the anisotropy of the solid may play a dominant role in the plastic flow of the solid as discussed in following sections.

4. SIMPLY SUPPORTED CIRCULAR PLATE OF ORTHOTROPIC MATERIAL

The determination of the load carrying capacities of circular plates has been of great interest in recent years (**7, 9, 10, 11, 12**). In 1953, Hopkins and Prager employed the concept of Tresca's yield condition and associated flow rules to evaluate the load carrying capacities of circular plates with various boundary conditions (**7**). In their investigation, the material was assumed to be isotropic and rigid-plastic.

Their method of approach will be extended in this section to analyze the plastic flow of simply supported circular plates of orthotropic material. This study will show the effects of anisotropy on the plastic flow of circular plate as well as illustrate the application of the modified Tresca's yield criterion for anisotropic materials.

Consider an orthotropic circular plate of radius R and thickness $2h$. Let r,θ,z be cylindrical coordinates. Passing through the center of the plate, the z-axis is normal to the middle plane of the plate and points downward. Let w denote the rate of deflection in the positive z-direction. The principal rates of curvature are

$$\left.\begin{aligned} K_r &= -\frac{d^2w}{dr^2} \\ K_\theta &= \frac{-1}{r}\frac{dw}{dr} \end{aligned}\right\}. \tag{6}$$

A transverse load of intensity p is applied in the following manner:

$$\begin{aligned} p &= \text{constant} \quad \text{for } 0 \leq r \leq a \\ p &= 0 \qquad\quad \text{for } a \leq r \leq R. \end{aligned}$$

For the rotational symmetrical loading described, the equation of equilibrium of a thin circular plate is

$$\frac{d}{dr}(rM_r) - M_\theta = rQ, \tag{7}$$

where M_r and M_θ are the principal bending moments and Q is the shearing force and equal to

$$Q = -\frac{1}{2\pi r}\int_0^r \int_0^{2\pi} p r dr d\theta. \tag{8}$$

Substituting the quantity Q from Eq. 8 into Eq. 7, the equation of equilibrium becomes

$$\frac{d}{dr}(M_r \cdot r) - M_\theta = -\int_0^r prdr. \tag{9}$$

For the type of loading considered here, the circular plate will become plastic first at the center where the two principal bending moments M_r and M_θ are equal.

In a circular plate whose principal axes of anisotropy at all points coincide with the axes of the coordinate system, the principal axes of anisotropy will coincide with the principal axes of stress during deformation. Therefore, the yield condition for a plate of the perfectly plastic

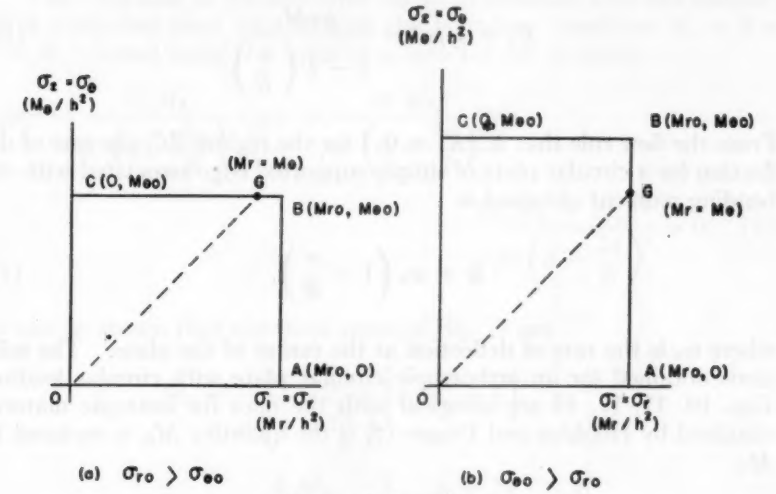

FIG. 2. Yield conditions.

material considered here can be represented by Fig. 2. For simplicity, the yield stress in tension is assumed to be equal to that in compression. Thus the yield condition shown in Fig. 2 may be considered as one in terms of bending moments.

Case 1: Yield Stress in the Radial Direction Being Greater than that in the Circumferential Direction $\sigma_{r_0} > \sigma_{\theta_0}$.

Consider the case that the yield stress in the radial direction is greater than that in the circumferential direction. Consequently, the limit moment M_{r_0} is greater than the limit moment M_{θ_0}. As shown in Fig. 2(*a*), point G where $M_r = M_\theta$ represents the stress state at the center of the plastically deformed plate. During plastic flow, the entire circular plate is in the regime G-C where $M_\theta = M_{\theta_0}$.

With the initial conditions $M_r = M_\theta$ at $r = 0$, the equation of equilibrium Eq. 9 can be solved for M_r. Replacing the quantity p by fp, it is found that

$$M_r = M_{\theta 0} - \tfrac{1}{6} fpr^2 \quad \text{for} \quad 0 \leq r \leq a. \tag{10}$$

$$M_r = M_{\theta 0} - \tfrac{1}{2} fpa^2 \left[1 - \frac{2a}{3r}\right] \quad \text{for} \quad a \leq r \leq R, \tag{11}$$

where f represents the multiply factor for the given load p to bring the plate to the flow limit. Using the fact that $M_r = 0$ at $r = R$, the total load at the flow limit is determined from Eq. 11,

$$P_1 = \pi fpa^2 = \frac{6\pi M_{\theta_0}}{3 - 2\left(\frac{a}{R}\right)}. \tag{12}$$

From the flow rule that $K_r : K_\theta = 0 : 1$ for the regime BC, the rate of deflection for a circular plate of simply supported edge associated with the bending moment obtained is

$$w = w_0 \left(1 - \frac{r}{R}\right), \tag{13}$$

where w_0 is the rate of deflection at the center of the plate. The solutions obtained for an orthotropic circular plate with circular loading, Eqs. 10, 11, 12, 13 are identical with the ones for isotropic material obtained by Hopkins and Prager (7) if the quantity M_{θ_0} is replaced by M_0.

Case 2: Yield Stress in the Circumferential Direction Being Greater than that in the Radial Direction $\sigma_{\theta_0} > \sigma_{r_0}$.

For the plate whose yield stress in the circumferential direction is greater than that in the radial direction, the limit moment M_{θ_0} is greater than the limit moment M_{r_0}. As shown in Fig. 2(*b*), the stress state at the center of the plate during plastic flow is represented by point G. Part of the plastically deformed plate is in the regime GB and part is in the regime BC. Solving the equation of equilibrium Eq. 9 in a similar manner, it is found that

$$\left.\begin{aligned} M_r &= M_{r_0} \\ M_\theta &= M_{r_0} + \tfrac{1}{2} fpr^2 \end{aligned}\right\} \tag{14}$$

for $0 \leq r \leq \rho_1$ where $\rho_1 = \sqrt{\dfrac{2(M_{\theta_0} - M_{r_0})}{fp}}$;

$$\left.\begin{aligned} M_\theta &= M_{\theta_0} \\ M_r &= M_{\theta_0} - \tfrac{1}{6} fpr^2 - \frac{2\rho_1}{3r}(M_{\theta_0} - M_{r_0}) \end{aligned}\right\} \tag{15}$$

for $\rho_1 \leq r \leq a$;

$$\left.\begin{aligned} M_\theta &= M_{\theta_0} \\ M_r &= M_{\theta_0} = \tfrac{1}{2} fpa^2\left[1 - \frac{2a}{3r}\right] - \frac{2\rho_1}{3r}[M_{\theta_0} - M_{r_0}] \end{aligned}\right\} \tag{16}$$

for $a \leq r \leq R$.

The total load at the flow limit can be determined after the quantity fp is evaluated from Eq. 16 with the boundary condition $M_r = 0$ at $r = R$. Substituting the value of ρ_1 into Eq. 16, it yields

$$(fp)^3 - \frac{12\,M_{\theta_0}}{a^2\left(3 - \dfrac{2a}{R}\right)}(fp)^2 + \frac{36\,M_{\theta_0}^2}{a^4\left(3 - \dfrac{2a}{R}\right)^2} fp - \frac{32(M_{\theta_0} - M_{r_0})^3}{a^4R^2\left(3 - \dfrac{2a}{R}\right)^2} = 0. \tag{17}$$

It can be shown that the three roots of Eq. 17 are

$$\left.\begin{aligned} fp_1 &= \frac{4\,M_{\theta_0}}{a^2\left(3 - \dfrac{2a}{R}\right)}\left[\cos\frac{\phi}{3} + 1\right] \\ fp_2 &= \frac{4\,M_{\theta_0}}{a^2\left(3 - \dfrac{2a}{R}\right)}\left[\cos\frac{\phi + 2\pi}{3} + 1\right] \\ fp_3 &= \frac{4\,M_{\theta_0}}{a^2\left(3 - \dfrac{2a}{R}\right)}\left[\cos\frac{\phi + 4\pi}{3} + 1\right] \end{aligned}\right\}, \tag{18}$$

where

$$\phi = \cos^{-1}\left[\frac{2a^2}{R^2}\left(3 - \frac{2a}{R}\right)\left(1 - \frac{M_{r_0}}{M_{\theta_0}}\right)^3 - 1\right].$$

By substituting the three roots given by Eq. 18 into second Eq. 16 and observing the boundary condition $M_{r_1} = 0$ at $r = R$, it is found that physically only the third root fp_3 is admissible.

Therefore, the multiplier f is found to be

$$f = \frac{4\,M_{\theta_0}}{pa^2\left[3 - \frac{2a}{R}\right]}\left[\cos\frac{\phi + 4\pi}{3} + 1\right]$$

provided that the ratio M_{r_0}/M_{θ_0} has a value between zero and unity.

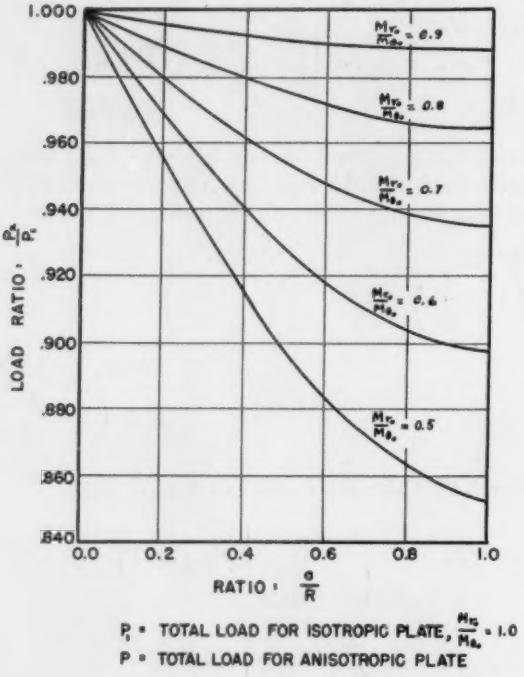

FIG. 3. Load ratios.

Using the multiplier f obtained, the total load at flow limit for an orthotropic plate whose yield stress in the circumferential direction is greater than that in the radial direction is

$$P_2 = \frac{4\pi\,M_{\theta_0}}{\left(3 - \frac{2a}{R}\right)}\left[\cos\left(\frac{\phi + 4\pi}{3}\right) + 1\right]. \tag{19}$$

From Eq. 19, the effects of the anisotropy on the total load at the flow limit can be evaluated and is illustrated in Fig. 3.

From the flow rules that $K_r : K_\theta = 1:0$ for the regime AB and $K_r : K_\theta = 0:1$ for the regime BC, the rate of deflection for circular plate with

simply supported edge associated with the bending moments evaluated is

$$\left.\begin{aligned} w &= w_0 \quad \text{for} \quad 0 \leq r \leq \rho_1 \\ w &= w_0 \frac{R}{R-\rho_1}\left(1-\frac{r}{R}\right) \quad \text{for} \quad \rho_1 \leq r \leq R \end{aligned}\right\} \tag{20}$$

In other words, a circular plate of orthotropic material whose yield stress in the circumferential direction is greater than that in the radial direction will deform into a shell in the shape of a truncated cone instead of a conical shell for the isotropic case (**7, 12**).

In summary, for orthotropic materials of which the yield stress in the circumferential direction is less than that in the radial direction, the load carrying capacity and deflection curve of a circular plate with circular loading and simple supported edge are similar to those for isotropic material. However, for orthotropic materials whose yield stress in the circumferential direction is greater than that in the radial direction, the load carrying capacity and deflection curve of a circular plate with circular loading and simply supported edge are different from those for isotropic material and depend on the degree of anisotropy as measured by the ratio M_{r_0}/M_{θ_0}. In other words, the anisotropy of a circular plate may affect its load carrying capacity and deflection curve, even though the parameter of anisotropy may not appear in the yield condition and associated flow rules.

5. ENLARGEMENT OF A CIRCULAR HOLE

The problem of the finite enlargement of a circular hole in a sheet has been treated by Taylor (**14**), Hill (**15**) and Prager (**5**). The sheet was assumed to be isotropic and perfectly plastic. Introducing the concept of Tresca's yield condition and associated flow rules, Prager showed that the problem could be solved in a simple and elegant way. In a similar manner, the enlargement of a circular hole in a sheet of orthotropic material will be discussed in this section. Such a study is to show the distinctive features of the plastic flow of anisotropic materials as well as to illustrate the application of the modified Tresca's yield criterion and associated flow rules.

Consider the enlargement of a circular hole in an infinite sheet of orthotropic material. Let a_0 be the initial radius of the hole, h_0 be the initial thickness of the sheet and ρ_1 be the radius of the elastic-plastic interface. σ_r and σ_θ are the principal stresses. The modified Tresca's yield criterion for a strained body whose principal axes of anisotropy coincide with the principal axes of stress during deformation is shown in Fig. 1. The stress state at the elastic-plastic interface is represented by point G where $\sigma_\theta = -\sigma_r > 0$. The associated flow rules for the regime

GD representing the stress state of the plastic region just inside the interface are

$$\dot{\epsilon}_r : \dot{\epsilon}_\theta : \dot{\epsilon}_z = -\sqrt{2}\sin\beta : \sqrt{2}\cos\beta : \sqrt{2}(\sin\beta - \cos\beta), \tag{21}$$

where

$$\dot{\epsilon}_r = \frac{\partial v}{\partial r}, \quad \dot{\epsilon}_\theta = \frac{v}{r}, \quad \dot{\epsilon}_z = \frac{\dot{h}}{h}$$

and

$$\beta = \tan^{-1}\left(\frac{\sigma_{\theta_0}{}^t}{\sigma_{r_0}{}^c}\right) = \tan^{-1}(n).$$

Since the radial velocity v vanishes at the interface $r = \rho_1$, it is found from the ratio of $(\dot{\epsilon}_r/\dot{\epsilon}_\theta) = -\tan\beta$ that $v = 0$ for the regime $G - D$. Consequently, all the strain rates are found to be equal to zero and the thickness of the sheet remains as h_0 in this zone. The equation of equilibrium in this rigid plastic region just inside the interface $r = \rho_1$ becomes

$$r\frac{\partial \sigma_r}{\partial r} + (\sigma_r - \sigma_\theta) = 0. \tag{22}$$

For the regime $C - D$, the yield condition is

$$\frac{\sigma_r}{(\sigma_{r_0}{}^c)} - \frac{\sigma_\theta}{(\sigma_{\theta_0}{}^t)} = -1. \tag{23}$$

Substituting the value of σ_θ from Eq. 23 into Eq. 22 and using the boundary condition $\sigma_\theta = -\sigma_r$ at $r = \rho_1$, the integration of Eq. 22 yields

$$\left.\begin{aligned}\sigma_r &= \frac{\sigma_{\theta_0}{}^t}{1-n}\left[1 - \frac{2}{1+n}\left(\frac{\rho_1}{r}\right)^{1-n}\right] \\ \sigma_\theta &= \frac{\sigma_{\theta_0}{}^t}{1-n}\left[1 - \frac{2n}{1+n}\left(\frac{\rho_1}{r}\right)^{1-n}\right]\end{aligned}\right\}. \tag{24}$$

The circumferential stress σ_θ as given by Eq. 24 decreases as r decreases, and becomes zero at

$$r = \rho_2 = \left(\frac{2n}{1+n}\right)^{\frac{1}{1-n}}\rho_1. \tag{25}$$

Therefore, the stress state at $r = \rho_2$ reaches the state represented by point D in Fig. 1. Furthermore, it can be shown that the stress state for the material inside the circle $r = \rho_2$ remains at point D (7). From

Table I, the flow rules corresponding to point D are

$$\dot{\epsilon}_r : \dot{\epsilon}_\theta : \dot{\epsilon}_z = [-\sqrt{2}\,(1-\lambda)\sin\beta - \lambda] : [\sqrt{2}\,(1-\lambda)\cos\beta] : [\sqrt{2}\,(1-\lambda)(\sin\beta - \cos\beta) + \lambda], \quad (26)$$

where $0 \leq \lambda \leq 1$. It follows that the sheet thickness h will no longer remain constant inside the circle $r = \rho_2$. Consequently, the equation of equilibrium becomes

$$r\frac{\partial}{\partial r}(h\sigma_r) + h(\sigma_r - \sigma_\theta) = 0. \quad (27)$$

Considering that inside the circle $r = \rho_2$, the stress state represented by point D in Fig. 1 is $\sigma_r = -\sigma_{r_0}{}^c$, $\sigma_\theta = 0$ and that the radial velocity $v = 0$ at $r = \rho_2$, the variation of the thickness of the sheet inside the circle $r = \rho_2$ is found from Eq. 27 to be

$$h = h_0\frac{\rho_2}{r}. \quad (28)$$

From the condition of incompressibility $\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = 0$, the radial velocity v inside the circle $r = \rho_2$ follows

$$v = 1 - \frac{r}{\rho_2}. \quad (29)$$

The strain rates can be evaluated from Eqs. 28 and 29, or

$$\dot{\epsilon}_r : \dot{\epsilon}_\theta : \dot{\epsilon}_z = -\frac{1}{\rho_2} : \frac{1}{r} : -\frac{1}{\rho_2} : \frac{2}{\rho_2} : -\frac{1}{r} \quad (30)$$

In computing the strain rates, ρ_2 is employed as a time parameter which increases monotonically.

Comparing Eqs. 26 and 30, the following relationship between r and ρ_2 is obtained

$$r = \frac{\sqrt{2}(1-\lambda)n + \lambda\sqrt{1+n^2}}{\sqrt{2}(1-\lambda)(n+1) + \lambda\sqrt{1+n^2}}\rho_2. \quad (31)$$

Since the plastic deformation is postulated to produce no volume change in the strained body, it follows

$$2\pi\int_r^{\rho_2} hrdr = \pi h_0(\rho_2^2 - r_0^2), \quad (32)$$

where the radius r_0 is the initial position of a material point and the radius r is its position at time ρ_2. Substituting the value of h from Eq.

28 into Eq. 32, one obtains

$$r = \frac{\rho_2}{2}\left[1 + \left(\frac{r_0}{\rho_2}\right)^2\right]. \tag{33}$$

Therefore, the movement of the material points within the circle $r = \rho_2$ is completely defined by Eqs. 31 and 33.

The initial position of a material point at radius r at time ρ_2 can be found to be at radius r_0 by placing the value of r from Eq. 31 into Eq. 33.

$$r_0 = \rho_2\left[1 - \frac{2\sqrt{2}(1-\lambda)}{\sqrt{2}(1-\lambda)(n+1) + \lambda\sqrt{(1+n^2)}}\right]^{\frac{1}{2}}. \tag{34}$$

In other words, a material point initially at radius r_0 as specified by Eq. 34 will move to the position at radius r at time ρ_2 as given by Eq. 33. The plastic flow at the point at radius r is governed by the flow rules Eq. 26 with the coefficient λ having a value which satisfies Eq. 34.

Case 1: $\sigma_{\theta_0}{}^t < \sigma_{r_0}{}^c$ *or* $n < 1$

For a material whose tensile yield stress in the circumferential direction is less than its compressive yield stress in the radial direction, the anisotropy parameter n is less than unity. Now vary the value of λ in Eq. 34 from unity toward zero. One finds that $r_0 = 0$ when λ assumes the value of

$$\lambda_0 = \frac{\sqrt{2}(1-n)}{\sqrt{2}(1-n) + \sqrt{1+n^2}}. \tag{35}$$

For $r_0 = 0$, r is found to be equal to $\frac{1}{2}\rho_2$ from Eq. 33. In other words, a pin hole of radius $r_0 = 0$ initially will expand into a hole of radius $r = \frac{1}{2}\rho_2$ at time ρ_2. At the boundary of the hole at time ρ_2, the plastic flow is governed by the following flow rules:

$$\dot{\epsilon}_r : \dot{\epsilon}_\theta : \dot{\epsilon}_z = [-\sqrt{2}(1-\lambda_0)\sin\beta - \lambda_0] : [\sqrt{2}(1-\lambda_0)\cos\beta] : [\sqrt{2}(1-\lambda_0)(\sin\beta - \cos\beta) + \lambda_0]. \tag{36}$$

Case 2: $\sigma_{\theta_0}{}^t > \sigma_{r_0}{}^c$ *or* $n > 1$

Now consider the material whose tensile yield stress in the circumferential direction is greater than its compressive yield stress in the radial direction. The anisotropy parameter n is greater than unity. Letting the value of λ in Eq. 34 vary from unity toward zero, one finds

that at

$$r_0 = \sqrt{\frac{n-1}{n+1}}\,\rho_2 \tag{37}$$

the coefficient λ is equal to zero. From Eq. 33 the radius r is found to be

$$r = \rho_3 = \frac{n}{n+1}\,\rho_2, \tag{38}$$

where the flow rules Eq. 21 govern. Namely, material points at radius r_0 initially as given by Eq. 37 will move to new positions at radius r as specified by Eq. 38 at time ρ_2.

For a circular hole of initial radius smaller than

$$r_0 = \rho_2\sqrt{\frac{n-1}{n+1}}$$

the material initially located within the circle of radius r_0 will have to form a third plastic zone inside the circle of radius ρ_3. In this zone, the plastic flow will be governed by the yield condition Eq. 23, the equation of equilibrium Eq. 27 and the flow rules Eq. 21.

Since the radial velocity v must be continuous at the boundary $r = \rho_3$ it is found from the ratio of $\dot{\epsilon}_r$ to $\dot{\epsilon}_\theta$ in Eq. 21 that inside the circle of radius ρ_3

$$v = \left(1 - \frac{\rho_3}{\rho_2}\right)\left(\frac{\rho_3}{r}\right)^n \tag{39}$$

in which ρ_2 is employed as a time parameter. Taking into consideration the condition of incompressibility $\dot{\epsilon}_r + \dot{\epsilon}_\theta + \dot{\epsilon}_z = 0$ the variation of the thickness within the circle of $r = \rho_3$ is found to be

$$\left(\frac{h}{h_0}\right) = \left(\frac{r}{\rho_3}\right)^{n-1} \cdot \left(\frac{h_3}{h_0}\right)^{\frac{1}{M}\left[\frac{\rho_2}{\rho_3} - \log\frac{\rho_2}{\rho_3} - \frac{1}{n+1}\left(\frac{r}{\rho_3}\right)^{n+1}\right]}, \tag{40}$$

where

$$M = \frac{n^2+n+1}{n(n+1)} - \log\frac{n+1}{n}.$$

In deriving Eq. 40, it is taken into consideration that the sheet thickness at $r = \rho_3$ is equal to $h_3 = \dfrac{(n+1)h_0}{n}$ from Eq. 28 at time $\rho_2 = \dfrac{n+1}{n}\,\rho_3$.

Combining the yield condition and the equation of equilibrium and using the boundary condition that $\sigma_r = -\sigma_{r_0}{}^c$ at $r = \rho_3$, the radial stress

σ_r in the third plastic zone is found to be

$$\frac{\sigma_r}{\sigma_{r_0}{}^c} = e^{NX}\left\{\frac{n}{n+1}\left[\log X - N(X-1) + \frac{N^2(X^2-1)}{2\cdot 2!} - \frac{N^3(X^3-1)}{3\cdot 3!} + \cdots\right] - e^{-N}\right\} \quad (41)$$

where

$$N = \frac{\log (h_3/h_0)}{M(n+1)} \quad \text{and} \quad X = \left(\frac{r}{\rho_3}\right)^{n+1}.$$

By evaluating a relationship similar to Eq. 32, the relation between the initial position r_0' and final position r of a material point in the third plastic zone at time ρ_3 is obtained.

$$\left(\frac{r}{\rho_3}\right)^{n+1} = 1 - \frac{1}{N}\log\left\{1 + \frac{n}{2}N\left[\frac{n^2-1}{n^2} - \left(\frac{r_0'}{\rho_3}\right)^2\right]\right\}. \quad (42)$$

Therefore, a hole of initial radius r_0 will be expanded into a hole of radius ρ_4 at time ρ_3

$$\rho_4 = \rho_3\left[1 - \frac{1}{N}\log\left\{1 + \frac{n}{2}N\left[\frac{n^2-1}{n^2} - \left(\frac{\rho_0}{\rho_3}\right)^2\right]\right\}\right]^{\frac{1}{n+1}} \quad (43)$$

However, in order to determine the radius ρ_4 completely, it is necessary to know the total force applied at the boundary of the expanded hole. By combining Eqs. 40 and 41,

$$P = (-2\pi r h\sigma_r)_{r=\rho_4}$$
$$= 2\pi\sigma_{r_0}{}^c h_0\rho_4 Y^{\frac{n-1}{n+1}}\left(\frac{h_3}{h_0}\right)^{\frac{1}{M}\left[\frac{n+1}{n} - \log\frac{n+1}{n} - \frac{1}{n+1}Y\right]}$$
$$\times e^{NY}\left\{e^N - \frac{n}{n+1}\left[\log Y - B(Y-1) + \frac{B^2(Y^2-1)}{2\cdot 2!} - \frac{B^3(Y^3-1)}{3\cdot 3!} + \cdots\right]\right\}, \quad (44)$$

where $Y = \left(\frac{\rho_4}{\rho_3}\right)^{n+1}$. Therefore, from Eqs. 25, 38, 43, and 44, the expansion of a circular hole of initial radius r_0' can be determined.

In summary, for an orthotropic material whose tensile yield stress in the circumferential direction is less than its compressive yield stress

in the radial direction, a circular hole in an infinite sheet can be forced to expand essentially in the same manner as in the case for an isotropic material, although the stress distributions are somewhat different from each other. However, for an orthotropic material whose tensile yield stress in the circumferential direction is greater than its compressive stress in the radial direction, the mode of plastic flow during the expansion of a circular hole is characteristically different from the isotropic case. In both cases, the stress distribution in the sheet shows dependence on its degree of anisotropy. Likewise, the yield condition and its associated flow rules for the problem considered are affected by the degree of anisotropy of the sheet.

6. CONCLUSION

By extending the concept of Tresca's yield criterion and associated flow rules for isotropic solids advanced by Prager, a modified Tresca's yield condition and associated flow rules for anisotropic materials were suggested. Such an approach was shown to be practical provided that the principal axes of stress coincide with the principal axes of anisotropy during deformation. Two problems, having known solutions for isotropic cases (**5, 7**), were given to illustrate the applicability of the modified Tresca's yield criterion and associated rules. Namely, the load carrying capacities of simply supported circular plates of orthotropic material were evaluated, and the enlargement of a circular hole in an infinite sheet of orthotropic material was analyzed. Otherwise, the analysis of these problems would be very much involved. For cases in which the principal axes of anisotropy do not coincide with the principal axes of stress, this approach loses its merit of solving problems in the plastic flow of anisotropic bodies simply. In this latter case, the analytical approach (**3, 6**) may prove to be more desirable.

From the results obtained in the two studies made, it was found that the anisotropy of a plastically deformed body may affect its stress distribution as well as its mode of deformation even though the parameter of anisotropy may not appear in the flow rules. Moreover, the effects of anisotropy on the plastic flow are shown to be dependent on the degree of anisotropy of the strained body. In other words, it is impossible to postulate the flow of a strained body of anisotropic material on the basis of its behavior if it were isotropic.

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HEAT TRANSFER IN TURBULENT PIPE FLOW *

BY

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ABSTRACT

Heat transfer to and from fully developed turbulent flow in a circular pipe and between parallel plates is analyzed.

The use of Pai's velocity and correlation distributions allows a consistent determination of the eddy diffusivity for heat $\epsilon_H = \alpha\epsilon_M$. The average of this ϵ_H distribution across the conduit is about one half of the value used in former investigations.

By application of Laplace's transform the partial boundary-value problems for temperature distribution in circular pipe and parallel plates are reduced to ordinary Sturm-Liouville problems. The corresponding eigenvalues and eigenfunctions are approximated by Ritz's method for different values of the Prandtl and Reynolds numbers. The temperature distribution and Nusselt number distribution along the walls follow in series form in terms of these eigenvalues and eigenfunctions.

With the classic hypothesis $\alpha = 1$ a new and better fitting to the experimental Nusselt number data for liquid metals is obtained. This analysis may be applied to any fluid in fully developed turbulent pipe flow.

INTRODUCTION

The analysis of the heat transfer in a turbulent flow, particularly from or to the walls limiting the fluid, requires the knowledge of the kinematical magnitudes characterizing the flow.

In the case of "forced flow" it is assumed that the velocity field, either laminar or turbulent, is independent of the temperature distribution and not perturbed by the small differences of density produced mainly by the differences of temperatures. To this case corresponds the so-called "fully developed" turbulent flow in a finite pipe in which there is a difference of pressures between its terminal cross sections.

The equation for the temperature distribution in a viscous flow is derived by consideration of a local energy balance. The equation for an incompressible flow is (1)[2]

$$\rho c \frac{DT}{Dt} = k\Delta T + \frac{\Phi}{J}, \tag{1}$$

where ρ is the density, c the specific heat, $\frac{DT}{Dt}$ the total derivative of T, k the thermal conductivity, Δ the operator of Laplace, and J the me-

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chanical equivalent of heat. The rate of dissipation of energy by viscous shear per unit time and unit volume is expressed by the Rayleigh dissipation function Φ. The absolute temperature at any point of a turbulent flow region has a mean value which shall be denoted by $\bar{T}$ and a fluctuation T' of zero mean value. Considering cartesian orthogonal coordinate axes the instantaneous velocity components are denoted by u_1, u_2, u_3, along the x_1, x_2, and x_3 axes, respectively. The velocity has mean components $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ and fluctuating components u_1', u_2', u_3'.

The measure δ of the time interval for the average is such as to give mean values independent of the time. These mean values are the quantities normally observed.

Replacing these quantities in the temperature distribution equation, taking mean values, and assuming mean steady heat flow it follows:

$$\rho c \sum_i \bar{u}_i \frac{\partial \bar{T}}{\partial x_i} = \sum_i \frac{\partial}{\partial x_i}\left(k \frac{\partial \bar{T}}{\partial x_i} - \rho c \overline{u_i' T'}\right), \quad (i = 1,2,3). \tag{2}$$

The last equation shows that the effect of the fluctuations is to add the eddy heat transfer terms $\rho c \overline{u_1' T'}$, $\rho c \overline{u_2' T'}$, $\rho c \overline{u_3' T'}$ to the heat transfer terms due to molecular conductivity.

In this analysis it will be considered that the distributions of mean velocity and velocity fluctuations are those of isothermal flow. The dissipation of energy by viscosity and the variation of conductivity, specific heat and viscosity with the temperature are here neglected. With the assumption of small temperature differences and considering the specific heat at constant pressure the following results may be applied to the heat transfer of compressible fluids.

It is assumed that the mean velocity vector has components along the x_1 axis only, that is, (4)

$$\bar{u}_1 = \bar{u}_1(x_1, x_2), \quad \bar{u}_2 = \bar{u}_3 = 0.$$

In the case of fully developed turbulent flow between infinite parallel walls which are perpendicular to x_2, the eddy heat transfer term $\overline{u_1' T'}$ is independent of x_1, $\overline{u_3' T'}$ is zero and $\dfrac{\partial^2 \bar{T}}{\partial x_1^2}$ may be considered to be negligible. Since the axial conduction is assumed very small as compared to the large axial convection, the term $\dfrac{\partial^2 \bar{T}}{\partial x_3^2}$ was also neglected.

The final equation for the present analysis is

$$\rho c \left[\bar{u}_1 \frac{\partial \bar{T}}{\partial x_1} + \frac{\partial}{\partial x_2}(\overline{u_2' T'})\right] = k \frac{\partial^2 \bar{T}}{\partial x_2^2}. \tag{3}$$

In the case of the circular pipe with cylindrical coordinates, the average velocity components are assumed to be (5)

$$\bar{u} = 0, \bar{v} = 0, \bar{w} = \bar{w}(r),$$
$$u' = u, v' = v, w' = w - \bar{w}$$

along r, θ and z, respectively.

For an axial symmetric distribution of temperatures the corresponding equation is

$$\rho c \left[\bar{w}(r) \frac{\partial \bar{T}}{\partial z} + \frac{1}{r} (\overline{u'T'}) + \frac{\partial}{\partial r} (\overline{u'T'}) \right] = k \left[\frac{1}{r} \frac{\partial \bar{T}}{\partial r} + \frac{\partial^2 \bar{T}}{\partial r^2} \right]. \quad (4)$$

2. FORMULATION OF THE BOUNDARY-VALUE PROBLEMS

Case of the Circular Pipe

By the momentum transfer theory (2, 3)

$$\overline{u'w'} = - \epsilon_M \frac{d\bar{w}}{dr}, \quad (5)$$

$$\overline{u'T'} = - \epsilon_H \frac{\partial \bar{T}}{\partial r}, \quad (6)$$

$$\epsilon_H = \alpha \epsilon_M(r) = - \alpha \frac{\overline{u'w'}(r)}{\left(\frac{d\bar{w}(r)}{dr} \right)}, \quad (7)$$

where α is the ratio of eddy diffusivities ϵ_H and ϵ_M.

With the usual designation: P_e is the Péclet number, $P_e = P_r \cdot R_e$ defined as the product of the Prandtl number, $P_r = \mu c/k$ and Reynolds number, $R_e = \dfrac{2a\bar{w}_m}{\nu}$; a is the radius of the pipe; $\mu = \nu\rho$ is the absolute viscosity; and $\bar{w}_m = \bar{w}(0)$ is the maximum mean flow velocity at the pipe axis. With Eq. 7, Eq. 4 is written

$$\frac{\partial}{\partial r} \left\{ r \left[\epsilon_H(r) + \frac{\nu}{P_r} \right] \frac{\partial \bar{T}}{\partial r} \right\} - r\bar{w}(r) \frac{\partial \bar{T}}{\partial z} = 0. \quad (8)$$

Considering the boundary conditions $\bar{w}(a) = 0$, $\overline{u'w'}(a) = 0$ (a being the radius of the circular pipe), Pai (4, 5) has found the form of a polynomial approximating $\bar{w}(r)$ which reduces to the parabolic distribution of Poiseuille when $\overline{u'w'}(r) \equiv 0$ and such that its radial derivative at the wall matches with the corresponding shear stress τ_0,

$$\bar{w}(\eta) = \bar{w}(0) f_1(\eta). \quad (9)$$

The distribution of $\overline{u'w'}(r)$ has also a polynomial form and it is easy to see that from Eqs. 21, 29, 52 of (5) follows

$$\overline{u'w'}(\eta) = \frac{\nu}{a} \bar{w}_m f_2(\eta), \tag{10}$$

where

$$\xi = \frac{z}{a}, \quad \eta = \frac{r}{a}, \tag{11}$$

$$f_1(\eta) = 1 + C_1\eta^2 + C_2\eta^{2n}, \quad C_1 = \frac{s-n}{n-1}, \quad C_2 = \frac{1-s}{n-1}, \tag{12}$$

$$f_2(\eta) = C_3\eta[1 - \eta^{2(n-1)}], \quad C_3 = 2\frac{n(s-1)}{(n-1)}. \tag{13}$$

As was defined by Pai (5), s is the ratio of the shearing stress τ_0 on the wall of the pipe for the turbulent flow to the corresponding τ_l on the wall for laminar flow with the same $\bar{w}_m$. In turbulent flow $s > 1$ for $s = 1$, Eqs. 9 and 10 reduce to the case of laminar flow with parabolic velocity profile and $\overline{u'w'} \equiv 0$. The value of n has to be determined from experimental data.

From Eq. 9 it follows

$$\tau_0 = -\mu\left[\frac{d\bar{w}(r)}{dr}\right]_{r=a} = -\mu\frac{\bar{w}_m}{a}f_1'(1), \quad \tau_l = \mu\frac{2\bar{w}_m}{a}. \tag{14}$$

Hence $s = -f'(1)/2$ and from the data given in Fig. 4 of (6) for $R_e = 50{,}000$, it results $s = 21.85$.

By inspection, it was found that the values $s = 21.85$, $n = 32$ for $f_1(\eta)$, $f_2(\eta)$ give a good agreement with the empirical functions represented in Figs. 3, 4 and 8 of (6).

Hence Eqs. 12 and 13 have the determination:

$$C_1 = -0.32742, \quad C_2 = -0.67258, \quad C_3 = 43.0445.$$

The temperature of the fluid at the walls is assumed constant, $\bar{T}(z,a) = \bar{T}_w$. If $\bar{T}_0$ is a mean of the inlet temperatures $\bar{T}(0, r)$ it is possible to change the dependent variable $\bar{T}(z,r)$ by means of the transformation:

$$U(\xi,\eta) = \frac{\bar{T}(z,r) - \bar{T}_w}{\bar{T}_0 - \bar{T}_w}. \tag{15}$$

Using Eqs. 11 and 15 and assuming a circular symmetric distribution of the inlet temperatures, because of the symmetry condition for the flow, a homogeneous boundary value problem for $0 \leq \eta \leq 1$, $\xi > 0$, can

be formulated as follows:

$$\frac{\partial}{\partial\eta}\left[A(\eta)\frac{\partial U}{\partial\eta}\right] - \frac{\partial U}{\partial\xi}B(\eta) = 0, \tag{16}$$

$$U(\xi,1) = \frac{\bar{T}(z,a) - \bar{T}_w}{\bar{T}_0 - \bar{T}_w} = 0, \tag{17}$$

$$U_0(\eta) = \lim_{\xi\to 0} U(\xi,\eta) = u^* U_0^*(\eta), \tag{18}$$

$$\left[\frac{\partial U(\xi,\eta)}{\partial\eta}\right]_{\eta=0} = 0, \tag{19}$$

where

$$A(\eta) = \eta[1 - \alpha P_r\varphi(\eta)], \quad \varphi(\eta) = -\frac{\epsilon_M(\eta)}{\nu} = \frac{f_2(\eta)}{f_1'(\eta)}, \tag{20}$$

$$B(\eta) = \frac{P_e}{2}\eta f_1(\eta). \tag{21}$$

Case of the Parallel Plates

Considering cartesian orthogonal coordinate axes, the equations of the pair of infinite parallel plates are $x_2 = -a$ and $x_2 = a$, respectively. The distance between the plates is $2a$.

Similarly to Eqs. 5, 6, and 7 and by assuming that the mean values of the turbulent fluctuations are functions of x_2 only,

$$\overline{u_1'u_2'} = -\epsilon_M(x_2)\frac{d\bar{u}_1}{dx_2}, \quad \overline{u_2'T'} = -\epsilon_H(x_2)\frac{\partial\bar{T}}{\partial x_2},$$

$$\epsilon_H(x_2) = \alpha\epsilon_M(x_2) = -\alpha\frac{\overline{u_1'u_2'}}{\dfrac{d\bar{u}_1}{dx_2}}.$$

As in the case of the circular pipe (see Eqs. 17, 25, and 47 of (4)) with symmetric flow, by assuming an inlet temperature distribution $\bar{T}(0,x_2)$ symmetric with respect to the plane $x_2 = 0$, a homogeneous boundary-value problem for $1 \le \eta = \frac{x_2}{a} \le 1$, $\xi = \frac{x_1}{a} > 0$ is obtained:

$$\frac{\partial}{\partial\eta}\left[A(\eta)\frac{\partial U}{\partial\eta}\right] - \frac{\partial U}{\partial\xi}B(\eta) = 0, \tag{22}$$

$$U(\xi,1) = \frac{\bar{T}(x_1,a) - \bar{T}_w}{\bar{T}_0 - \bar{T}_w} = 0, \tag{23}$$

$$U_0(\eta) = \lim_{\xi \to 0} U(\xi,\eta) = u^* U_0^*(\eta), \tag{24}$$

$$U(\xi,-1) = 0, \tag{25}$$

where

$$A(\eta) = 1 - P_r \alpha \varphi(\eta), \tag{26}$$

$$\varphi(\eta) = -\frac{\epsilon_M(\eta)}{\nu} = \frac{f_2(\eta)}{f_1'(\eta)}, \tag{27}$$

$$B(\eta) = \frac{P_e}{4} f_1(\eta). \tag{28}$$

In this case the Reynolds number is

$$R_e = \frac{4a\bar{u}_{1m}}{\nu}. \tag{29}$$

By fitting the polynomials $f_1(\eta)$, $f_2(\eta)$ to Laufer's (7) experimental data for turbulent flow in a channel with $R_e = 40.200$, Pai (4) has found $s = 11.06$ and $n = 16$, so that Eqs. 12 and 13 have the determination:

$$f_1(\eta) = 1 - 0.3293\eta^2 - 0.6707\eta^{32}, \quad f_2(\eta) = 22.1486\eta[1 - \eta^{30}]. \tag{30}$$

3. SOLUTION OF THE BOUNDARY VALUE PROBLEMS

Case of the Circular Pipe

By application of the Laplace transformation (8) defined by

$$\mathfrak{L}\{U(\xi,\eta)\} = \int_0^\infty e^{-p\xi} U(\xi,\eta) d\xi = \omega(\eta,p),$$

the partial boundary value problem of Eqs. 16, 17, 18 and 19 will be transformed in an ordinary differential problem, the solution of which would lead, by the inverse transformation $\mathfrak{L}^{-1}\{\omega(\eta,p)\} = U(\xi,\eta)$ to the solution of the first. Applying $\mathfrak{L}$ to Eqs. 16, 17 and 18 and using a well known identity and the convolution theorem and the condition of Eq. 18, a new boundary value problem for the self-adjoint ordinary differential equation

$$L[\omega(\eta)] = [A(\eta)\omega'(\eta)]' - pB(\eta)\omega(\eta) = -B(\eta)U_0(\eta), \tag{31}$$

with the boundary conditions

$$\omega'(0) = 0, \quad \omega(1) = 0, \tag{32}$$

is formulated.

By the form of Eqs. 12, 13, 20 and 21, $A(\eta)$, $A'(\eta)$ and $B(\eta)$ are continuous and $A(\eta) \geq 0$, $B(\eta) \geq 0$ for $0 \leq \eta \leq 1$. The function $A(\eta)$

(Eq. 20) is zero for $\eta = 0$ and $A(1) = 1$; hence $\eta = 1$ is a "regular" point and the eigenvalues λ_m of the Sturm-Liouville problem are simple. From Eq. 21, $B(\eta) \geq 0$, $(0 \leq \eta \leq 1)$ hence these eigenvalues λ_m are real. The non-homogeneous Sturm-Liouville problem has a solution if the parameter $\lambda = -p$ is not an eigenvalue of the homogeneous problem (with $-B(\eta)U_0(\eta) \equiv 0$). As the λ_m's form a sequence of positive real numbers and as $\omega(\eta,p)$ is considered in a half plane of positive abscissa, then $\lambda = -p \neq \lambda_m$ and the non-homogeneous problem has a unique solution:

$$\omega(\eta,p) = -\sum_{m=0}^{\infty} \frac{a_m}{p + \lambda_m} \omega_m(\eta), \tag{33}$$

where

$$a_m = \int_0^1 [-B(\eta)U_0(\eta)]\omega_m(\eta)d\eta. \tag{34}$$

The system of eigenfunctions $\{\omega_m(\eta)\}$ is complete, in the sense of approximation in the mean, and is orthonormal relative to the weight function $B(\eta)$ over $[0,1]$:

$$e_{ik} = \int_0^1 B(\eta)\omega_i(\eta)\omega_k(\eta)d\eta = \begin{cases} 0, & i \neq k \\ 1, & i = k. \end{cases} \tag{35}$$

By application of $\mathfrak{L}^{-1}$ to Eq. 33 and by the convolution theorem, which requires that $U_0(\eta)$ be twice continuously differentiable, the solution $U(\xi,\eta)$ of Eqs. 16, 17, 18 and 19 is obtained as:

$$U(\xi,\eta) = \sum_{m=0}^{\infty} b_m e^{-\lambda_m \xi}\omega_m(\eta), \tag{36}$$

$$b_m = \int_0^1 B(\eta)U_0(\eta)\omega_m(\eta)d\eta. \tag{37}$$

It can be proved that the series of Eq. 36 is convergent and may be differentiated once with respect to η and twice with respect to ξ, since $U_0(\eta)$ is bounded. Hence, Eqs. 16, 17, 18 and 19 have a unique solution expressed by Eq. 36. The necessary condition, Eq. 18, of continuity at the boundary is also satisfied.

Considering mean steady heat flow in a semi-infinite pipe, the distribution $U_0(\eta)$ is being taken as almost constant at the entrance $(\xi = 0)$. In order to maintain the condition of continuity at the boundaries it is necessary that $U_0(1) = 0$. We assume here, instead of a strict constant distribution, that is, $U_0^*(\eta) = 1$, $(0 \leq \eta < 1)$, the distribution represented by the trigonometrical series approximating the square $U_0^*(\eta) = 1$, $-1 < \eta < 1$, $U_0^*(1) = 0$. This series has to be the one corresponding to the even function

$$U^*(\eta) = U^*(\eta + 2m) = 1, \quad m \text{ even},$$
$$U^*(\eta) = U^*(\eta + 2m) = -1, \quad m \text{ odd},$$

considered only in $0 \leq \eta \leq 1$, together with its integrals, for instance, the integral in Eq. 37 defining b_m.

The local heat flux in the fluid at the wall is

$$q(z) = k \left(\frac{\partial \bar{T}}{\partial r} \right)_{r=a} \tag{38}$$

where a is the radius of the pipe, and the local heat transmission coefficient (9) is given by

$$h(z) = \frac{q(z)}{\bar{T}_w - \bar{T}_{mm}(z)} = k \frac{\left(\frac{\partial \bar{T}}{\partial r} \right)_{r=a}}{\bar{T}_w - \bar{T}_{mm}(z)}, \tag{39}$$

where $\bar{T}_{mm}(z)$ is the "mixed mean temperature" defined as the weighted mean of $\bar{T}(r,z)$ relative to $rw(r)$, which by Eqs. 10 and 11, is

$$\bar{T}_{mm}(\xi) = \frac{\int_0^1 \bar{T}(\xi,\eta)\eta f_1(\eta)d\eta}{\int_0^1 \eta f_1(\eta)d\eta}. \tag{40}$$

From Eqs. 15, 40 and 39 it follows

$$h(\xi) = -\frac{k}{a} \frac{\left(\frac{\partial U(\xi,\eta)}{\partial \eta} \right)_{\eta=1}}{U_{mm}(\xi)}, \tag{41}$$

$$U_{mm}(\xi) = \frac{\int_0^1 U(\xi,\eta)\eta f_1(\eta)d\eta}{\int_0^1 \eta f_1(\eta)d\eta}. \tag{42}$$

The local Nusselt number is defined as

$$N_u(\xi) = \frac{2a}{k} h(\xi). \tag{43}$$

By means of Eqs. 41, 42 and 36, we obtain

$$N_u(\xi) = -2 \frac{\int_0^1 \eta f_1(\eta)d\eta \cdot \sum_{m=0}^{\infty} \left[b_m \left(\frac{d\omega_m(\eta)}{d\eta} \right)_{\eta=1} e^{-\lambda_m \xi} \right]}{\sum_{m=0}^{\infty} \left[b_m \left(\int_0^1 \omega_m(\eta)\eta f_1(\eta)d\eta \right) e^{-\lambda_m \xi} \right]}. \tag{44}$$

Then the asymptotic value of the Nusselt number is

$$N_u(\infty) = \lim_{\xi \to \infty} N_u(\xi) = -2 \frac{\omega_0'(1) \int_0^1 \eta f_1(\eta) d\eta}{\int_0^1 \omega_0(\eta) \eta f_1(\eta) d\eta}. \tag{45}$$

Hence $N_u(\infty)$ is characterized only by the first eigenfunction $\omega_0(\eta)$.

Case of the Parallel Plates

Applying the Laplace transformation to Eqs. 22, 23, 24 and 25 as before, and by Eq. 24, the following boundary-value problem is formulated:

$$L[\omega(\eta)] = [A(\eta)\omega'(\eta)]' - pB(\eta)\omega(\eta) = -B(\eta)U_0(\eta) \tag{46}$$

with boundary conditions

$$\omega(-1) = 0, \quad \omega(1) = 0. \tag{47}$$

Equations 26, 27, 28 and 30 show that $A(\eta)$, $A'(\eta)$, $B(\eta)$ are continuous symmetric non-negative functions in $-1 \leq \eta \leq 1$.

As in the previous section, the non-homogeneous Sturm-Liouville problem posed by Eqs. 46 and 47 has a unique solution of the form

$$\omega(\eta,p) = -\sum_{m=0}^{\infty} \frac{a_m}{p + \lambda_m} \omega_m(\eta), \tag{48}$$

where

$$a_m = \int_{-1}^{1} [-B(\eta)U_0(\eta)]\omega_m(\eta)d\eta. \tag{49}$$

The corresponding system of eigenfunctions, which is called here again $\{\omega_m(\eta)\}$ is complete and orthonormal relative to the weight function $B(\eta)$ over $[-1, 1]$:

$$\int_{-1}^{1} B(\eta)\omega_i(\eta)\omega_k(\eta)d\eta = \begin{cases} 0, & i \neq k \\ 1, & i = k. \end{cases} \tag{50}$$

The function $B(\eta)$ is given by Eq. 28.

Applying $\mathfrak{L}^{-1}$ to $\omega(\eta,p)$ (Eq. 46) and by the convolution theorem, it follows that

$$U(\xi,\eta) = \sum_{m=1}^{\infty} b_m e^{-\lambda_m \xi} \omega_m(\eta), \tag{51}$$

where

$$b_m = \int_{-1}^{1} B(\eta)U_0(\eta)\omega_m(\eta)d\eta. \tag{52}$$

As in the case of the circular pipe, Eq. 51 is the unique solution of the homogeneous problem posed by the parabolic Eq. 22 with boundary conditions 23, 24, and 25. According to the definition of "mixed mean temperature,"

$$\bar{T}_{mm}(\xi) = \frac{\int_{-1}^{1} \bar{T}(\xi,\eta) f_1(\eta) d\eta}{\int_{-1}^{1} f_1(\eta) d\eta},$$

the heat-transfer coefficient is given in the same form by Eq. 41 and:

$$U_{mm}(\xi) = \frac{\int_{-1}^{1} U(\xi,\eta) f_1(\eta) d\eta}{\int_{-1}^{1} f_1(\eta) d\eta}.$$

Defining the Nusselt number by Eq. 43, it follows that

$$N_u(\xi) = -2 \frac{\int_{-1}^{1} f_1(\eta) d\eta \sum_{m=0}^{\infty} [b_m \omega_m'(1) e^{-\lambda_m \xi}]}{\sum_{m=0}^{\infty} \left[b_m \left(\int_{-1}^{1} \omega_m(\eta) f_1(\eta) d\eta \right) e^{-\lambda_m \xi} \right]}. \tag{53}$$

Then, the asymptotic Nusselt number is:

$$N_u(\infty) = \lim_{\xi \to \infty} N_u(\xi) = -2 \frac{\omega_0'(1) \int_{-1}^{1} f_1(\eta) d\eta}{\int_{-1}^{1} \omega_0(\eta) f_1(\eta) d\eta}. \tag{54}$$

4. COMPUTATION OF THE SOLUTION

The Computation of Eigenvalues and Eigenfunctions

The calculation of the eigenvalues and eigenfunctions of the homogeneous differential problems of Section 3 can be accomplished by the application of the direct method of Ritz.

The problem of determining the solution of the homogeneous boundary value problem for the equation

$$L[\omega(\eta)] = 0, \quad a \leq \eta \leq b,$$

where L is a differential self-adjoint operator defined by Eq. 31 or Eq. 46, is equivalent to the problem of determining the extremal of a quadratic functional

$$F[\omega(\eta)] = \int_a^b A(\eta)[\omega'(\eta)]^2 - \lambda B(\eta)[\omega(\eta)]^2 d\eta, \tag{55}$$

in a class D_L of functions which satisfy the boundary conditions (Eqs. 31 and 32 or Eqs. 46 and 47).

The method of Ritz consists of the construction of the sequence of functions

$$\omega_m(\eta) = \sum_{i=0}^{m} a_i \psi_i(\eta) \quad (m = 0,1,2 \ldots), \tag{56}$$

where the Ritz "coordinate vectors" $\psi_i(\eta)$ in a Hilbert space H constitute a complete and linearly independent system in H, and are chosen from a set $D_L \subseteq H$.

The substitution of Eq. 56 into Eq. 55 provides a function $F = F[a_1,a_2, \ldots a_m,\lambda]$, for each m, whose extremal condition is expressed by a homogeneous system of linear equations in the a_i's. This system has a non-trivial solution if its determinant is equal to zero. This condition of compatibility is a polynomial equation of degree m in λ: $\mathcal{P}(\lambda) = 0$. Since the problem is self-adjoint, $\mathcal{P}(\lambda) = 0$ has m real solutions, $\lambda_i^{(m)}$, which are the "approximation of order m" for the first m eigenvalues λ_i. By introduction of each $\lambda_i^{(m)}$ in the linear system and with the "normalization condition" (Eq. 35 or 50), the corresponding solution $\{a_i^{(m)}\}$ $(i = 1,2, \ldots m)$ is determined.

Since all the eigenvalues are real, that is $L[\omega(\eta)]$ is definite positive, Ritz's method provides sequences $\{\lambda_i^{(m)}\}$, $\{\omega_i^{(m)}(\eta)\}$ convergent to the eigenvalues λ_i and eigenfunctions $\omega_i(\eta)$ as it has been proved (**10, 11**).

Case of the Circular Pipe

The system of polynomials

$$\psi_i(\eta) = 1 - \eta^{i+2}, \quad i = 0,1,2, \ldots \tag{57}$$

is complete in the sense of approximation in the mean, satisfies the boundary conditions of Eqs. 32 and is linearly independent because its Gram's determinant is different from zero. By Eqs. 55, 56 and 57 the linear homogeneous system is

$$\frac{\partial F[a_1,a_2, \ldots a_m,\lambda]}{\partial a_i} = \sum_{j=0}^{m} a_j\{I_{ij} - \lambda M_{ij}\} = 0, \tag{58}$$

$$I_{ij} = J_{ij} + K_{ij} \tag{59}$$

$$J_{ij} = \int_0^1 \eta[(2 + i)\eta^{1+i}][(2 + j)\eta^{1+j}]d\eta = \frac{(2 + i)(2 + j)}{4 + i + j} \tag{60}$$

$$K_{ij} = -\alpha P_r L_{ij}, \quad L_{ij} = \int_0^1 (2 + i)(2 + j)\eta^{3+i+j}\varphi(\eta)d\eta \tag{61}$$

$$M_{ij} = \frac{P_e}{2} N_{ij}, \quad N_{ij} = \int_0^1 \eta f_1(\eta)(1 - \eta^{2+i})(1 - \eta^{2+j})d\eta. \tag{62}$$

From Eqs. 12, 13, 20 and 61 it follows that

$$L_{ij} = (2 + i)(2 + j) \int_0^1 \frac{(1 - \eta^{2(n-1)})\eta^{3+i+j}}{2\dfrac{C_1}{C_2} + 2n\dfrac{C_2}{C_2}\eta^{2(n-1)}} d\eta, \tag{63}$$

and by Eqs. 12 and 62,

$$N_{ij} = \frac{1}{2} + \frac{1}{6+i+j} - \frac{1}{4+j} - \frac{1}{4+i} + C_1\left[\frac{1}{4} - \frac{1}{8+i+j} - \frac{1}{6+j} - \frac{1}{6+i}\right] + C_2\left[\frac{1}{2n+2} + \frac{1}{2n+6+i+j} - \frac{1}{2n+4+i} - \frac{1}{2n+4+j}\right]. \tag{64}$$

The polynomial equation of degree m:

$$\varphi(\lambda) = det|I_{ij} - \lambda M_{ij}| = 0$$

has m real solutions $\lambda_0^{(m)}$, $\lambda_1^{(m)}$, $\lambda_2^{(m)}$, . . . , $\lambda_m^{(m)}$ which are upper bounds of the corresponding eigenvalues λ_0, λ_1, λ_2, . . . λ_m. For each one of the λ_p each homogeneous system

$$\sum_{j=0}^{m} \alpha_j\{I_{ij} - \lambda_p^{(m)} M_{ij}\} = 0 \quad (i,j,p = 0.1, \ldots m), \tag{65}$$

where $\alpha_j = \dfrac{a_j}{a_m}$, $\alpha_m = 1$ has a solution $\alpha_1^{(p)}$, $\alpha_2^{(p)}$, . . . $\alpha_{m-1}^{(p)}$, $\alpha_m^{(p)} = 1$. From Eqs. 21, 56, 57 and 62 and the orthonormality condition of Eq. 35, the coefficient a_m is characterized by

$$a_m^{(p)} = + \left[\sum_{i=0}^{m}\sum_{j=0}^{m} \alpha_i^{(p)}\alpha_j^{(p)} M_{ij}\right]^{-\frac{1}{2}}. \tag{66}$$

An indication of the approximation given by the method of Ritz is expressed by the convergence to zero of the mean orthogonality error:

$$e_{p,q}^{(m)} = a_m^{(p)} a_m^{(q)} \frac{P_e}{2}\left[\sum_{i=0}^{m}\sum_{j=0}^{m} \alpha_i^{(p)}\alpha_j^{(q)} M_{ij}\right] \to 0, \quad p \neq q \tag{67}$$

for each p, q and increasing values of m.

Case of the Parallel Plates

If we consider a symmetric distribution of inlet temperatures $U_0(\eta)$ the problem reduces to the strip $0 \leq \eta \leq 1$, $\xi > 0$, and the boundary condition of Eq. 25 is transformed into

$$\left[\frac{\partial U(\xi,\eta)}{\partial \eta}\right]_{\eta=0} = 0$$

whereby the coefficients in Eq. 51 become

$$b_m = \int_0^1 B(\eta) U_0(\eta) \omega_m(\eta) d\eta. \tag{68}$$

The system of polynomials given by Eq. (57) is also admissible in this case because it satisfies the new boundary conditions.

Similar to the case of the circular pipe discussed above, one obtains a system such as Eq. 58 with coefficients

$$J_{ij} = \int_0^1 (2+i)\eta^{1+i}(2+j)\eta^{1+i} d\eta = \frac{(2+i)(2+j)}{3+i+j} \tag{69}$$

$$K_{ij} = -\alpha P_r L_{ij}, \quad L_{ij} = \int_0^1 (2+i)(2+j)\eta^{2+i+j}\varphi(\eta) d\eta \tag{70}$$

$$M_{ij} = \frac{P_e}{4} N_{ij}, \quad N_{ij} = \int_0^1 f_1(\eta)(1-\eta^{2+i})(1-\eta^{2+i}) d\eta. \tag{71}$$

By Eqs. 27, 30 and 70 it follows that

$$L_{ij} = (2+i)(2+j) \int_0^1 \frac{(1-\eta^{2(n-1)})\eta^{2+i+j}}{2\dfrac{C_1}{C_3} + 2n\dfrac{C_2}{C_3}\eta^{2(n-1)}} d\eta \tag{72}$$

and by Eqs. 30 and 71

$$\begin{aligned} N_{ij} = &\left[1 + \frac{1}{5+i+j} - \frac{1}{3+j} - \frac{1}{3+i}\right] \\ &+ C_1\left[\frac{1}{3} + \frac{1}{7+i+j} - \frac{1}{5+j} - \frac{1}{5+i}\right] \\ &+ C_2\left[\frac{1}{2n+1} + \frac{1}{2n+5+i+j} - \frac{1}{2n+3+i} - \frac{1}{2n+3+j}\right]. \end{aligned} \tag{73}$$

The process of calculation is the same as indicated for the circular pipe.

Calculation of Nusselt Number Distribution in Thermal Entrance Regions for the Case of Liquid Metals

The liquid metals region can be considered (12) to be defined by $0.002 < P_r < 0.03$. For $R_e = 50{,}000$ the corresponding Péclet numbers are $100 < P_e < 1500$. For this interval of Péclet numbers, experimental heat-transfer data for fully developed turbulent flow of liquid metals in circular pipes are available (13, 14, 15). The results of the calculations for $m = 2$, $\alpha = 1$ and $R_e = 50{,}000$ and five different Prandtl numbers in the liquid metal region are summarized in the following. Table I contains the coefficients defined by Eqs. 59, 60, 61,

TABLE I.

$L_{00} = -50.260$	$L_{01} = -56.503$	$L_{02} = -58.856$
$L_{10} = -56.503$	$L_{11} = -66.213$	$L_{12} = -70.994$
$L_{20} = -58.856$	$L_{21} = -70.994$	$L_{22} = -77.766$
$N_{00} = 0.153007$	$N_{01} = 0.175940$	$N_{02} = 0.189201$
$N_{10} = 0.175940$	$N_{11} = 0.203914$	$N_{12} = 0.220477$
$N_{20} = 0.189201$	$N_{21} = 0.220477$	$N_{22} = 0.239318$
$J_{00} = 1.000000$	$J_{01} = 1.200000$	$J_{02} = 1.333333$
$J_{10} = 1.200000$	$J_{11} = 1.500000$	$J_{12} = 1.71429$
$J_{20} = 1.333333$	$J_{21} = 1.71429$	$J_{22} = 2.000000$

P_r	0.002	0.010	0.014	0.020	0.030
P_e	100	500	700	1000	1500
$a_0^{(0)}$	35.3	2.1443	1.9373	1.6467	1.3679
$a_1^{(0)}$	−16.6	−2.2721	−2.1482	−2.0102	−1.8945
$a_0^{(1)}$	1.6361	1.6160	2.1188	2.9707	2.8292
$a_1^{(1)}$	2.5037	−2.4847	−2.9231	−3.6643	−3.5389
$a_0^{(2)}$	35.3	0.8145	0.8183	0.8248	0.8327
$a_1^{(2)}$	−16.6	−1.7838	−1.7873	−1.7931	−1.8002
$a_0^{(0)}$	0.729	0.4499	0.3756	0.3289	0.2994
$a_1^{(0)}$	−0.343	−0.4765	−0.4166	−0.4015	−0.4147
$a_2^{(0)}$	0.021	0.2098	0.1939	0.1997	0.2189
$a_0^{(1)}$	8.140	3.6972	2.5258	1.7888	1.4922
$a_1^{(1)}$	−12.456	−5.6848	−3.4846	−2.2065	−1.8665
$a_2^{(1)}$	4.975	2.2879	1.1921	0.6022	0.5274
$a_0^{(2)}$	15.031	7.0623	6.0276	5.0573	4.1928
$a_1^{(2)}$	−33.279	−15.4670	−13.1650	−10.9941	−9.0643
$a_2^{(2)}$	18.797	8.6708	7.3659	6.1312	5.0352
λ_0	0.13865	0.03858	0.03135	0.02580	0.02137
λ_1	0.756	0.206	0.172	0.143	0.118
λ_2	2.195	0.568	0.441	0.357	0.292
$N_u(\infty)$	5.4	7.15	7.56	8.153	9.04

62, 63, 64 and 65. The first three approximate eigenvalues and the first three approximate eigenfunctions with the "mean normality error" $\epsilon_{pp}^{(2)} - 1$ and "mean orthogonality error" $\epsilon_{pq}^{(2)}$, $(p,q = 0,1,2)$ were computed in (16). The asymptotic Nusselt numbers, $N_u(\infty)$, were calculated by application of Eq. 45. The distribution of local Nusselt numbers in the thermal entrance region is given by Eq. 44. Since

$$\omega_m(\eta) = a_0^{(m)}(1 - \eta^2) + a_1^{(m)}(1 - \eta^3) + a_2^{(m)}(1 - \eta^4), \quad (m = 0,1,2)$$

the values $\omega_m'(1)$ follow. Considering $U_0(\eta) = 1, 0 \leq \eta < 1$, $U_0(1) = 0$, the coefficients b_m as defined by Eq. 37 are given by

$$b_m = \frac{P_e}{2}\int_0^1 \eta f_1(\eta)\omega_m(\eta)d\eta = \frac{P_e}{2}\sum_{i=0}^{2} a_i^{(m)}A_i,$$

where $A_i = \int_0^1 \eta f_1(\eta)(1 - \eta^{2+i})d\eta = N_{i,-\infty}$, as given by Eq. 64.

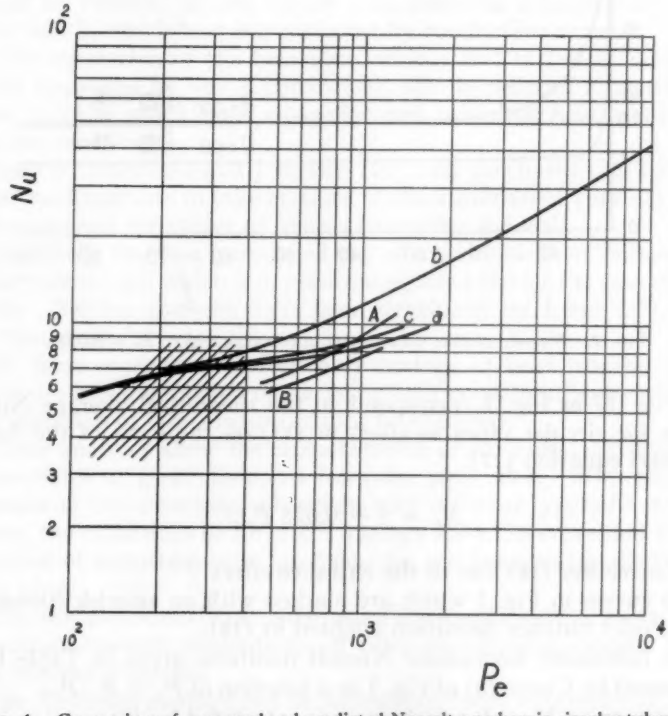

FIG. 1. Comparison of measured and predicted Nusselt numbers in circular tubes. A*—Gilliland, Musser and Page (heating), Refs. (13, 15). B*—Gilliland, Musser and Page (cooling), Refs. (13, 15). a—Calculated asymptotic Nusselt number $N_u(\infty)$. b*—Theoretical curve of Refs. (15, 17) for constant temperature at the wall. c*—Calculated average, Nusselt number. Doody and Younger, Refs. (14, 15). $(*P_e = 2P_rR_e\int_0^1 \eta f_1(\eta)d\eta,\ P_r = c\mu/k,\ R_e = 2a\bar{v}(0)/\nu.)$

5. COMPARISON OF EXPERIMENTAL AND THEORETICAL NUSSELT NUMBERS FOR LIQUID METALS

Average heat transfer coefficients in circular tubes with uniform wall temperature were measured ($0 \leq \xi \leq 90$) by Gilliland, Musser and Page (13) (mercury) and by Doody and Younger (14) (mercury with sodium addition).

According to Lubarsky (15) the last data present a great scatter, but are included in part to indicate a tendency in the lower Péclet range ($100 < P_e < 500$).

Curves *A* and *B* in Fig. 1 represent mean lines through the data of (13) and the cross-hatched area the data of (14) as given in (15).

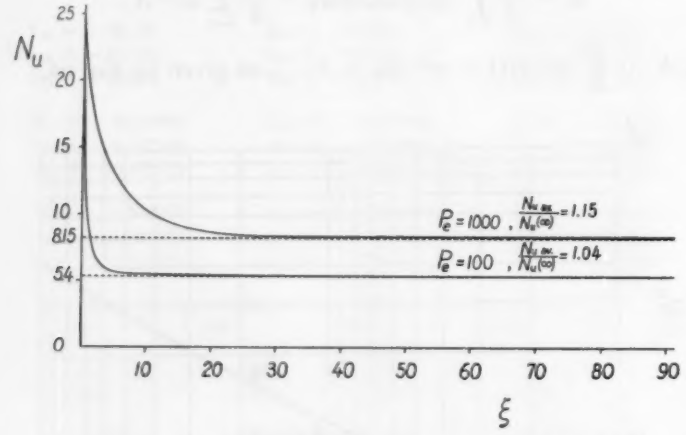

FIG. 2. Nusselt number distribution in entrance region, $R_e = 50{,}000$.

Curve (*b*) in Fig. 1 correspond to the theoretical average Nusselt number for circular pipes as given in (13) on the basis of the Seban-Shimazaki equation (17).

$$N_u = 5 + 0.025\, P_e^{0.8}$$

and a correction (18) due to the entrance effect.

The curves in Fig. 1 which are marked with an asterisk correspond to the Péclet number definition adopted in (15).

The calculated asymptotic Nusselt numbers, given in Table I, are represented by Curve (*a*) of Fig. 1 as a function of $P_e = R_e \cdot P_r$.

The average Nusselt numbers were calculated by integration in the range $0 \leq \xi \leq 100$ of the local Nusselt numbers as given in Fig. 2. It can be observed that the new theoretical curve (*c*) of Fig. 1 is in better agreement with the experimental data.

Background and Related Investigations

Osborne Reynolds (19) was the first who inferred a dependency between the transference of heat and momentum in turbulent flow. In the case $P_r = 1$, $\alpha = 1$, the postulate of a constant ratio between shearing stress and heat transfer leads to Reynolds formula expressing the heat transmission coefficient at the wall as proportional to the skin-friction coefficient and the Reynolds number.

Dittus and Boelter (20) and Colburn (21) gave formulas which in fact contain empirical corrections to the Reynolds formula taking into account the difference between temperature and velocity distribution near the wall.

Since the analysis of Reynolds holds under special conditions which are not generally attained, it cannot be applied directly to the heat transfer between solids and fluids.

For such cases, extensions of the analogy for $P_r \neq 1$ have been developed by Prandtl (3) and Taylor (22), assuming a laminar layer close to the wall in which heat is transferred by conduction alone.

This separation of the field into laminar and turbulent layers was further improved by von Kármán (23), who introduced a transition or buffer layer in which both molecular and turbulent heat transactions were assumed.

Boelter, Martinelli and Jonassen (24), and Reichardt (25) have suggested modifications to take account of the variation of viscosity due to the transversal derivative of temperature near the wall.

Martinelli (26) has introduced the effect of molecular diffusivity in the turbulent core, which is of essential significance for the case of liquid metals. Similar analyses have been developed by Lyon (27), Seban and Shimazaki (17).

All these extensions of Reynolds analogy of heat transfer to skin friction are based implicitly on an asymptotic consideration by which the axial variation of temperature cannot be obtained.

These analyses allow the representation of the radial distribution of temperatures at great distances from the pipe inlet. With particular emphasis on the importance of regions with different types of heat transactions, the extensions of Reynolds analogy have circumvented the consideration of boundary value problems for the temperature distribution in the pipe.

This was particularly plausible at a time in which more exact knowledge about relevant magnitudes of fully developed turbulent flow in pipes was not available.

The Reynolds analogy can be formulated more rigorously (3, 25) by departing from the axial Reynolds equation and the temperature equation.

Prandtl (3) and Reichardt (25) have proved that the analogy is strictly realized not only if $P_r' = \alpha P_r = 1$ but also if the fluid contains

heat sources of intensity proportional to the constant axial component of the gradient of pressures.

The calculations based on the extensions of the analogy are usually compared, however, with measurements of heat transfer from fluids in conduits in which no sources of heat are present.

The more exact approach based on the partial differential equation for the temperature has been considered first by Latzko (28) for turbulent pipe flow with the 1/7th power law of velocity distribution. Since he neglected the molecular conduction up to the wall, considering only turbulent eddy convection, his calculations apply only in the neighborhood of $P_r = 1$.

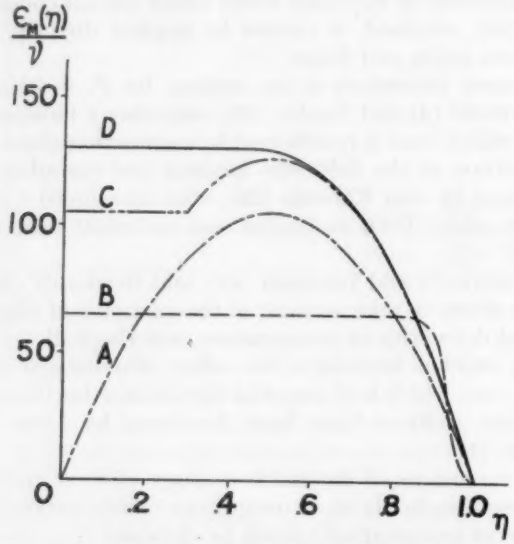

FIG. 3. Different momentum diffusivities, $R_e = 50{,}000$. Curve A—Refs. (32, 26) (logarithmic profile). Curve B—Distribution of $\frac{\epsilon_M(\eta)}{\nu}$ as given by Eqs. 12, 13 and 20. Curve C—Ref. (34). Curve D—Ref. (33).

Some authors have considered modified formulations leading to dependences $\alpha = \alpha(P_r)$, (29), where P_r is the Prandtl number and $\alpha = \alpha(P_e)$ (30), where P_e is the Péclet number of the flow.

Recently Schleicher and Tribus (31) have constructed solutions of the temperature equation by considering an empirical modification of the dependence $\alpha = \alpha(P_r)$, (29) with the object of a better fitting with the experimental data.

The low values of $\alpha < 1$, which (29) indicates for liquid metals, were considered (31) to be in agreement with the fact that the theoretical predictions of Martinelli (26) and Lyon (27) based on $\alpha = 1$ give

values of the local heat transmission coefficient and Nusselt numbers which are higher than the experimental ones. But it is essential to remember that $\epsilon_H = \alpha\epsilon_M$, and that higher theoretical values of the Nusselt number may be expected, not only if α is large but also if ϵ_M is generally large.

In many analyses (**26, 32, 33, 34**), some of them based on extensions of Reynolds analogy and predicting heat transfer rates higher than the experimental data, have been used distributions $\epsilon_M(\eta)$ which across the pipe have higher values than those used here for $R_e = 50{,}000$. (See Fig. 3.)

The present analysis is not based on a separation of the flow field into different layers, and when the parameters n, s in Pai distributions are taken as $n = 0$, $s = 1$ it reduces to the case of laminar flow treated by Graetz (**35**) and Nusselt (**9**).

Other solutions, for liquid metals, describing especially the thermal entrance region, were given by Poppendiek (**36**) and Harrison (**37**).

The turbulent eddy convection and the longitudinal conduction are neglected as compared with the radial conduction. The assumption about the axial conduction is also used in (**31, 38**) and here.

With consideration of a rectangular velocity distribution, Harrison (**37**) has calculated different solutions for the temperature with and without the axial heat conduction term. By inspection he concludes that the effect of the axial heat conduction term as compared with the axial convection is not significant for $P_e \geq 40$.

The same lower limit of applicability, $40 \leq P_e$ will be considered provisionally in the following. This is the only limitation of the present analysis which may be applied to any fluid in fully developed turbulent flow.

6. CONCLUSIONS

With the consideration of the polynomial mean velocity and correlation distributions given by Pai as solutions of Reynolds equations for turbulent flow, the temperature distribution and heat transfer rates for fully developed turbulent flow in a circular pipe or between parallel plates has been calculated.

It has been shown that in related analytical investigations the eddy diffusivity for momentum, ϵ_M, along the transversal coordinate is higher than in the present analysis. Since the eddy diffusivity for heat is $\epsilon_H = \alpha\epsilon_M$, this may explain that generally the calculated values of heat transfer coefficients and Nusselt number were higher than the experimental ones.

Some authors have adjusted the values of α with a factor, such that an agreement with the experimental results was reached (**31**).

The use of Pai's distributions allows a consistent determination of the eddy diffusivity for heat ϵ_H with a minimum of assumptions. With

the classic hypothesis $\alpha = 1$, a better fitting to the experimental heat-transfer data for liquid metals has been obtained.

As compared to analyses based on extensions of Reynolds analogy of heat transfer to skin-friction, the present study contains no asymptotic considerations and the axial variation of temperature and Nusselt numbers in the thermal entrance region are obtained simultaneously.

When the parameters in Pai's distribution reduce to $n = 0$ and $s = 1$ the present analysis reduces to the case of laminar flow treated by Graetz (35) and Nusselt (9).

This method of calculation of temperature and Nusselt number distribution may be applied to any fluid in fully developed turbulent flow with the only provisional limitation $P_e \geq 40$.

The basis for further calculation in a larger range of Prandtl and Reynolds numbers is given and it remains to be seen if the hypothesis $\alpha = 1$ and the lower limit $P_e \geq 40$ need further adjustment to fit the experimental data or not.

The consideration of higher approximations for the calculation of higher eigenvalues and eigenfunctions by means of computer devices will be of advantage for a final over-all application of the solution.

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Electronic Vehicle Control for Traffic Accident Problem.—An electronic system that provides automatic control of vehicles on the highway offers "the ultimate solution" to the problem of traffic accidents that now claim more than 100 lives daily, Dr. V. K. Zworykin, Honorary Vice-President of the Radio Corporation of America, told the Highway Research Board at its annual meeting.

Addressing more than 1000 highway engineers and administrators attending the conference of the board, a part of the National Academy of Sciences, Dr. Zworykin called for cooperation among highway engineers, automobile manufacturers, and the electronics industry in the development of a complete electronic highway control system. He pointed out that the basic elements of one such system have already been developed by RCA and were recently tested and demonstrated successfully in cooperation with the State of Nebraska in a 300-ft. experimental highway installation at Lincoln.

A description of the system tested in Nebraska was given to the board by L. E. Flory, who led a research team in its development. Mr. Flory demonstrated the operating principles of the system with the aid of a specially-developed simulator which duplicated all of the functions of a full-size highway installation and presented the results on an oscilloscope screen for display to the audience over a closed-circuit television system.

The RCA system, as described by Mr. Flory, consists basically of a series of electronic detector units buried in and along the highway, and a guidance cable buried in the center of a traffic lane along the length of the road. The detector units, in which signals are generated by the passage of vehicles, are used to activate various systems of warning lights and can eventually be combined with simple receivers and automatic equipment in vehicles to control speed and braking. The guidance cable radiates a constant signal which is employed in conjunction with a car receiver and automatic steering equipment to keep a vehicle centered in its traffic lane.

The RCA scientists pointed out that the system includes provision for a radio "tail warning" that extends any desired distance behind each passing vehicle to warn or control following cars equipped with receivers, or to operate warning lights along the roadside.

Stating that the basic elements of the experimental system had proven successful in the Nebraska tests, Dr. Zworykin said:

"The next logical step is the construction of an enlarged test facility which would permit testing the various features of the system at full scale and at normal speeds with completely controlled experimental vehicles. Such a facility should be at least two miles in length and be constructed with means for performing tests needed for determining the optimum length and shape of the warning 'tail,' the optimal dimensions and installations for the detector units, the comparative performance of different types of detectors, and the proper functioning of other components of the complete system."

The Franklin Institute

ANNUAL REPORT

of the Board of Managers for the Year 1957

Summary

The Board of Managers, at the beginning of this 135th year of The Franklin Institute, is pleased to review the twelve months just past. The By-Laws of the Institute require that an annual report be presented to the members at the annual meeting. This report will examine the events of 1957 in relation to the Institute's primary purpose—"promotion of the mechanic arts"—a term which we interpret in the light of today's technology as "the physical sciences and engineering."

The efforts of The Franklin Institute during 1957, as in many years past, have been divided among three broad programs: (1) to interest children and the general public in science; (2) to serve scientists and engineers in the pursuit of their professions; and (3) to contribute something of its own to the advance of science through research.

The first program—to interest school-age children and adults in science —encompasses the many activities of the Museum, as well as the lectures delivered at Institute meetings. Three major exhibits were opened in the Museum during 1957, covering the newest developments in computation, model railroading, and nuclear power reactors. Attendance for the year set new records. In the Planetarium the program of the International Geophysical Year was featured in several shows; and at the time Sputnik was put into its orbit the current show fortunately dealt with earth satellites, which made it easy to satisfy the public interest in the scientific background of this exciting event. The 1957 lectures covered a wide range of subjects and admirably served their purpose of keeping members of the Institute up-to-date in many branches of science and engineering.

The second program—to serve professional scientists and engineers—is carried out through the JOURNAL, *the Committee on Science and the Arts, the Library, and sponsorship of special meetings and symposia. Thus the Institute serves not only its immediate community, but also a world-wide scientific community. The Institute sponsored three major technical sym-*

posia in 1957: on computer techniques, air pollution and air traffic control. These were attended by specialists from all over the United States.

The third program—research—is pursued at The Franklin Institute Laboratories for Research and Development, at the Bartol Research Foundation of The Franklin Institute, and at the Biochemical Research Foundation, for which the Institute is trustee. The Laboratories carry on research in such subjects as air pollution, data processing, bioengineering, electronics, air traffic control, structure of metals, bearings, missile components, and nuclear reactor design. One phase of the work at Bartol in 1957 concerned cosmic-ray studies in connection with the program of the International Geophysical Year. The Biochemical Research Foundation continued its studies on cancer, with particular emphasis on the search for inhibitors that might be effective in the control of cancer.

The success of these programs depends upon the interest of the more than 7000 Institute members, the ability and enthusiasm of the nearly 600 members of the staff, and the generosity of the individual and corporate Friends of Franklin, who contribute toward the support of the Institute.

The first three sections of the main body of this report describe in some detail the three mainstreams of the Institute's activities. A fourth section tells how the Institute functions.

The Board of Managers, in presenting this Annual Report, is pleased with the progress evidenced in all of the Institute's programs for "promoting the mechanic arts." We appreciate the fine team-work of the staff that made this progress possible. We also express our thanks to those who have contributed their thoughts, their time and their money toward the fulfillment of our aims. We look forward in 1958 to a continuation and an expansion of the present activities.

Respectfully submitted,
By order of the Board of Managers
S. WYMAN ROLPH
President

January 15, 1958

Science Education

In 1957, our country was jolted out of its belief in the superiority of American technology. The result has been a public clamor for more scientists, with better training. The solution to the problem is two-fold: (1) to capture good minds for science at an early age and (2) to inform the general public what the needs of science are. The Institute, through its Museum and its lectures, has been attacking the problem from both these avenues of approach for many years.

MUSEUM

The Science Museum of the Institute is a center of attraction for thousands of school children and adults, eager to see the latest wonders of science displayed in an attractive, easily understood way. The combined Museum and Planetarium attendance during November set an all-time record for a one-month period.

The popularity of the Museum has increased steadily in the past few years, as shown by these statistics:

Museum Attendance

	Children*	Adults	Total
1953	203,698	85,704	289,402
1954	209,306	89,088	298,394
1955	219,765	92,248	312,013
1956	238,207	98,063	336,270
1957	263,305	114,304	377,609

* Up to and including those of senior high school age.

One reason for this popularity is that new exhibits are frequently opened, to reflect recent technical developments, as illustrated on pages 230 and 232.

New Exhibits Opened During 1957

The Computation Room

An exhibit of the history of computation, presented by the Remington Rand Univac Division of the Sperry Rand Corporation, January 22, 1957. It features a view of the Institute's Univac Computing Center.

The Telephone Room

The exhibit that had been seen by 1,402,969 visitors during the preceding five years, was brought up-to-date during the summer of 1957 by the Bell Telephone Company of Pennsylvania.

The Electric Battery

An animated display that explains what makes an electric storage battery work, on loan from the Electric Storage Battery Company, opened September 17, 1957.

EDUCATION DIVISION

A concerted effort to interest people in science is made by the Education Division. In 1957, 173,167 school children were taken on guided tours of the Museum facilities during school hours. In addition to these regular tours, special lecture-demonstrations were given on fire prevention, liquid air and the science of music.

For after-school hours, the Division sponsored several workshops, during both summer and winter, in which students were supervised in do-it-yourself projects in basic physical science, photography, astronomy, electronics and model making. During 1957, 102 young people attended these workshops, conducted in a new classroom designed for this purpose.

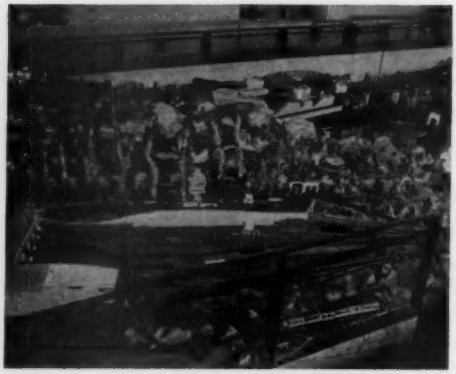
Among the special events for students is the annual *Science Fair,* sponsored jointly by the Institute and the Philadelphia Inquirer. The one held in April 1957 had 364 exhibits by Delaware Valley students. Six sessions of *career forums,* in chemistry, electrical engineering and mathematics, sponsored jointly by the Institute and several technical societies, attracted 2163 junior and senior high school students. This contribution to science education was praised in a telegram from the President of the United States in November.

In October, Professor Eric M. Rogers, of Princeton, delivered the annual James Mapes Dodge Lecture for young people, on "Drops, Splashes and Atoms." It was so popular that several hundred students were turned away.

The Education Division program depends upon the close cooperation of the public, private and parochial schools in Philadelphia and the surrounding ten counties. Without this cooperation, its program would not be effective.

Engineers of the future making drawbridges from two milk cartons, one pencil and one piece of string, in a Winter Workshop.

New Exhibits Opened During 1957



The Pennreading Railroad

"The Poor Richard Line," with 1000 feet of track, is one of the most advanced HO gauge model railroads in the world. Sponsored by the Pennsylvania Railroad, The Reading Company, and the Eastern Model Railroad Manufacturers, it was opened October 28, 1957.

"Coast Guard Rescue"

Large mural by Robert Paul Tosa, presented November 25, 1957, by Vice-Admiral A. T. Richmond of the U. S. Coast Guard for display in the Marine Section.

"From Atoms to Kilowatts"

A dynamic exhibit showing how electric power is produced by the use of a nuclear reactor. Sponsored by Leeds & Northrup Company, it opened November 19, 1957.

PLANETARIUM

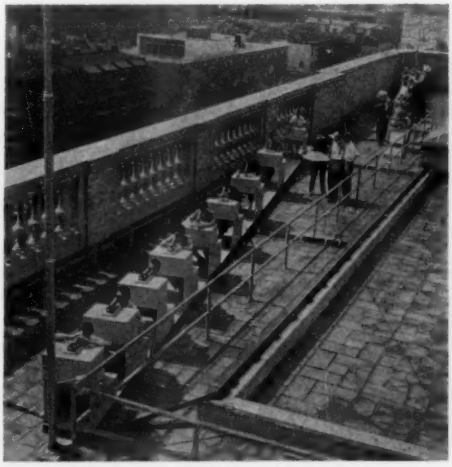
One of the most popular attractions in the Museum, the Planetarium continues to offer exciting instruction in the oldest of sciences, astronomy. In addition to the regular Planetarium shows covering a variety of subjects, a course of nine evening lectures on "Astronomy for Everyone" was attended by about 70 persons.

Planetarium Shows During 1957

Subject	Months
Eyes in the Sky	January
**South Pole Summer*	February
Our Mr. Sun	March
The End of the World	April, May and August
**Midsummer Night's Dream*	June
Moon—Target for Tomorrow	July
Reasons for the Seasons	September
**The Russian Satellite and Project Vanguard*	October and November
The First Christmas	December

* New in 1957.

Starting in June, the evening Planetarium shows were enlarged and designated as "Sky Tours." With the launching of the Russian satellites and the attendant interest on the part of the public, the "Satellite" show drew especially large crowds.



The Institute's official IGY "Moonwatch Station" for visual tracking of the earth satellites went into operation in October.

LECTURES

To interest adult laymen and specialized scientists who wish to keep abreast of fields other than their own, the Institute offers evening lectures on scientific subjects, in a semi-popular vein. All lectures are delivered by experts in the fields they discuss; questions from the audience are encouraged.

Meetings During 1957

	Date	Title	Speaker
S*	January 16	*Reality, Imagery and Fantasy*	Dr. W. F. G. Swann
J	January 30	*Modern Traffic Control for 1957*	Mr. Lionel M. Rogers
J	February 6	*Electronic Weather Prediction*	Mr. P. H. Kutschenreuter Mr. Henry P. Adams
F	February 14	*Computing Is Fun*	Dr. John W. Mauchly
JS	February 20	*The SAGE System and Its Implementation*	Brig. Gen. S. T. Wray Mr. Robt. Bright, Jr.
J	February 27	*Revolution in Color Printing*	Mr. Don Nibbelink
J	March 6	*How the Univac is Used in Physics and Engineering*	Mr. Donald B. Houghton
S	March 20	*Experiments in Television Over Telephone Cable Facilities*	Mr. C. Raymond Kraus
JS	April 17	*Farm Programs and Food Prices*	Dr. Wm. I. Myers
J	May 1	*The International Geophysical Year*	Dr. Serge A. Korff
S	May 15	*From Tesla Until Tomorrow—The Magic of Making Light*	Dr. Samuel G. Hibben
F	June 6	*Operation Moonwatch*	Dr. I. M. Levitt
M	September 25	*The House of Magic* presented by the General Electric Company	Mr. Kenneth G. Patrick
S	October 16	*Deliberate Speed*	Dr. Hugh Stott Taylor
F	October 28	Preview of The Model Railroad	—
J	October 30	*Electronics in Telephone Switching Systems*	Mr. H. N. Seckler
J	November 6	*Long Range Planning—Modern Panacea or Ancient Flop?*	Dr. Felix A. Kalinski
S	November 20	*Safe Atoms*	Dr. C. Rogers McCullough
J	December 4	*Exploratory Research in Astronautics*	Dr. Morton Alperin
S	December 18	*Science and a Rational World Security System*	Col. Richard S. Leghorn

* F—"Science is Fun" Meeting.
J—Joint Meeting with local technical society.
M—Special Meeting for Members.
S—Stated Meeting.

Serving Scientists

The Institute has much to offer scientists and engineers in the pursuit of their professions. Its JOURNAL publishes original technical material; the Committee on Science and the Arts rewards outstanding achievement; its Library is available for reference work; and its symposia offer specialists an opportunity to exchange ideas with others in their field.

JOURNAL

In its 132nd year the JOURNAL published five Institute lectures, eight general interest papers and thirty-seven technical papers in fields ranging from strength of materials through mathematics, physics, electronics and chemistry, to earth satellites. An important feature is the book review section, giving authoritative opinions of new technical books.

Distributed in all 48 states of the United States and in over fifty foreign countries, the JOURNAL is the Institute's most widely known contribution to science. In June, circulation was 5051; due to a change in the By-Laws effective in July, whereby members no longer receive free subscriptions, circulation dropped to 3938 in December.

Monograph No. 3 of the JOURNAL series was published in April. Titled "Automatic Coding," it is the proceedings of a symposium held in February at the Institute. By the end of the year, sales had reached 1270. Monograph No. 2, "Earth Satellites as Research Vehicles," enjoyed continued popularity as a result of public interest in the IGY program.

COMMITTEE ON SCIENCE AND THE ARTS

Throughout the year this Committee, composed of approximately 70 members of the Institute, painstakingly reviews pioneering work in many fields of science and technology. Upon its recommendation, the Institute awards medals and certificates of merit for outstanding achievement, at impressive Medal Day ceremonies. These awards, which have been presented to eminent scientists of many nationalities, are world-renowned.

At the 1957 Medal Day ceremonies in October, eleven individuals, one company and one association were honored, as follows:

Medal	Recipient	Subject
Cresson	Willard Frank Libby	Radiocarbon dating
Cresson	Reginald James Seymour Pigott	Engineering accomplishments
Cresson	Robert A. Watson-Watt	Pulsed radar in United Kingdom
Longstreth	John Bertrand Johnson	Thermal noise
Levy	David Barnard Steinman	Paper on Mackinac Bridge
Henderson	Association of American Railroads	Railway engineering
Wetherill	Warren W. Carpenter	Telephone Automatic Message Accounting
Wetherill	The Martin Company	Bonded structures
Brown	Pier Luigi Nervi	Reenforced concrete
Clamer	William G. Pfann	Zone refining of metals
Ballantine	Robert Morris Page Leo Clifford Young	Pulsed radar in United States
Franklin	Hugh Stott Taylor	Heterogeneous catalysis

Dr. S. Wyman Rolph, President of the Institute, congratulates 1957 recipients of awards. Seated (left to right): Dr. Hugh Taylor, Dr. Robert Page, Mr. William Faricy (who accepted the award for the Association of American Railroads), and Dr. Willard Libby. Standing: Dr. Rolph, Mr. Warren Carpenter, Dr. John Johnson, Mr. G. G. Willey (who accepted the award for The Martin Company), Dr. David Steinman, Mr. Reginald Pigott, Mr. Leo Young, Mr. William Pfann, and Dr. Robert Watson-Watt.

SYMPOSIA

Realizing the importance—and the difficulty—of keeping abreast of latest developments in rapidly expanding fields, the Institute sponsors technical symposia to afford specialists an opportunity to discuss their problems with fellow experts. Three such meetings were arranged by the Laboratories in 1957, on these subjects:

Automatic Coding, a two-day symposium attended by 300 in February, covered the newest programming techniques for electronic computers and data-processing machines.

Particulate Emission, a one-day meeting in October, attended by 100, was the second annual symposium devoted to air pollution. It stressed the problems associated with flyash, soot, and dusts.

Airways Modernization Board, a three-day affair in December, attended by 350 persons, brought together interested segments of the aviation industry to discuss the functions of this newly created national board for air traffic control.

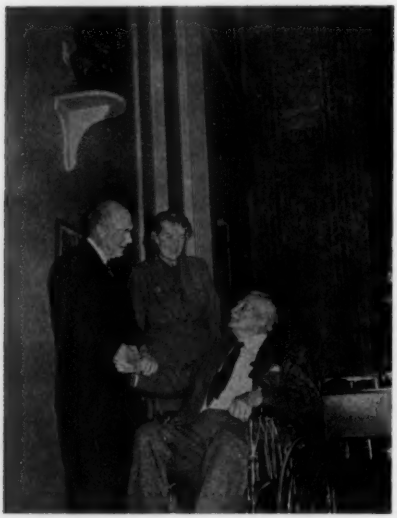
RESA

The fourth year of The Franklin Institute Branch of the Research Society of America was an unusually successful one. A new policy of luncheon meetings brought twelve speakers to the Institute. A high percentage of the 139 members of the staff who belong to RESA attended these meetings.

LIBRARY

The Library served 10,841 people during 1957. Of this number, 5990 made personal use of its collections, while 4851 were taken care of through telephone calls. There were 2590 new books added during the year, bringing the total to 159,000. The collection of scientific periodicals is one of the most complete in this area. Its worth was enhanced in 1957 by the addition of 22 new journals, so that at the end of the year 1522 were being received.

Worth noting is the acquisition of several rare volumes, among which are Archimedes' "Tetragonismus" (1503); Marcus' "De proportione motus" (1640); Schopper's "De omnibus illiberalibus sive mechanicis artibus" (1574); and Varignon's "Project d'une nouvelle mechanique" (1687).



In November, a bronze bust of The Honorable George Wharton Pepper, for whom Pepper Hall, the main Library reading room, is named, was unveiled with a brief ceremony attended by Mr. Pepper, Miss Agnes Yarnall, the sculptor, and Institute officials.

Research

The conduct of research is today the biggest single activity of the Institute. It overshadows the traditional activities of science education and furnishing aid to the professional man. The research program is carried out principally in The Franklin Institute Laboratories for Research and Development; but important segments of the program are conducted by the Bartol Research Foundation and the Biochemical Research Foundation.

LABORATORIES

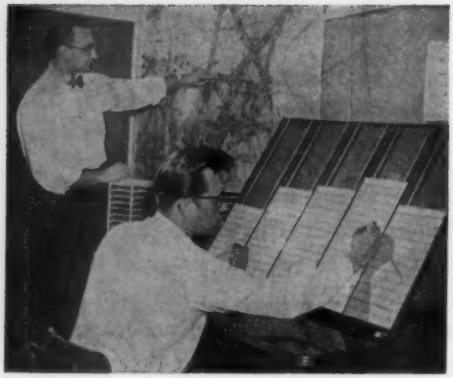
FOR RESEARCH AND DEVELOPMENT

The Franklin Institute Laboratories continued to grow during 1957, their twelfth year, as the statistics on page 20 indicate. The scope remained essentially the same as the year before, with projects distributed among these five divisions: chemistry and physics, electrical engineering, mechanical engineering, nuclear engineering, and solid state physics.

The work of the Laboratories has been made possible by outside financial support, usually through contracts having specific objectives. In reviewing the activities and achievements of the past year, it is convenient to enumerate a few of the more than 200 projects.

Public Service Projects

The pioneer work on *air pollution,* supported by the American Petroleum Institute, was extended by operation of a mobile air pollution laboratory in the Los Angeles area. The long-path infrared absorption cell used in this work made possible the identification of one of the major links in the series of reactions that cause smog.

Work on municipal engineering problems increased. Studies were conducted to learn the fundamental nature of frost action on underground pipes and to evaluate the utility of a gravity meter for the detection of wash-outs beneath city streets.

Studying air traffic patterns in the vicinity of a heavy air terminal.

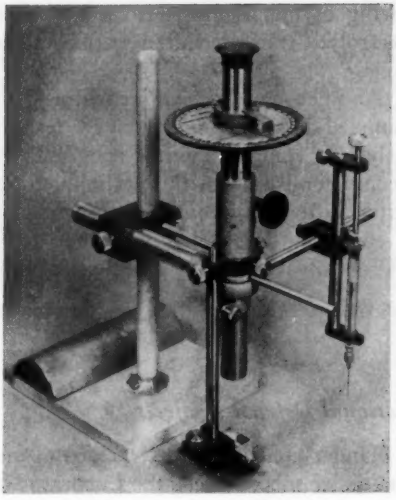
The long-range studies of *air traffic control* were continued for several government agencies. A project was started for the newly created Airways Modernization Board on an operations analysis of air traffic demand and delays in the New York area.

In *bioengineering,* a completely transistorized model of the electronic cane for the blind was built under the auspices of the Department of Health, Education and Welfare, for further field testing of this device. The Pew Foundation supported the development of a "Fibrascope," which is a flexible endoscope utilizing very fine glass fibers for the optical examination of internal organs of the human body. A feasibility study of what products blind workers might manufacture was started for the Pennsylvania State Council for the Blind.

Developments for Industry

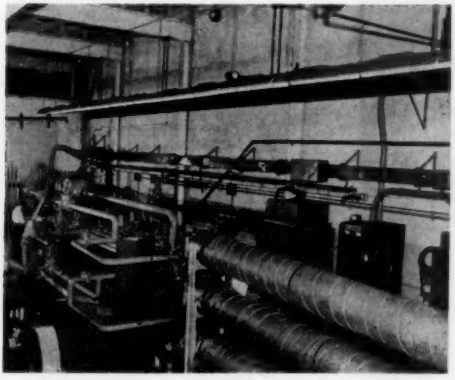
In accordance with one of the objectives stated in the Institute's charter—"the promotion and encouragement of manufactures"—the Laboratories engage in the development of new devices and new or improved manufacturing methods. Among accomplishments of this kind during 1957 were:

Adhesive bonding as applied to aircraft structures;
Automatic machine to manufacture envelopes for special mailing purposes;
Bearing for a roll in a paper mill;
Pilot plant for saturating fiber paper;
Wrinkle tester for cotton cloth.
Transistorized liquid density measuring instrument for quality control in a chemical process;



Special apparatus for measuring contact angle of droplet of H_2O over a surface, to determine its readiness for adhesive bonding.

A large share of the effort in nuclear engineering was devoted to research and development for Atomic Power Development Associates in connection with the Enrico Fermi power reactor to be operated by the Detroit Edison Company. This is the first sodium-cooled, fast-breeder power reactor to be built.



High pressure loop for studies in fluid mechanics and heat transfer.

Extensive analytical and experimental studies are being made on this project, with considerable attention being devoted to thermal stresses induced by gamma heat generation and temperature transients in the reactor coolant. New concepts have been developed for limiting the degree of

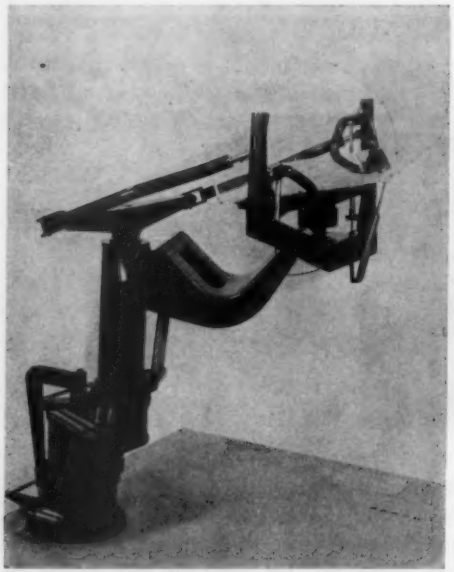
bowing of fuel elements caused by thermal gradients in the reactor core. Electrolytic tracer techniques are being employed in studies of reactor coolant mixing.

Another development benefiting government as well as industry is a "sampled-data-analog-computer" technique. It has been applied to the analysis of complex thermal transients in a nuclear power plant. In general, a computer based on this technique, designed primarily for the solution of partial differential equations, extends the capability of the analog computer to complex, distributed-parameter problems, which heretofore were considered solvable only by a digital computer.

National Defense Projects

Under contracts with various agencies of the Federal Government and also under subcontracts, the Laboratories are engaged in missile and weapons research and development. Most of these projects are classified.

Mention can be made of the development and construction of a Helicopter Flight Simulator Platform. It is intended to serve both for pilot training and also as a research tool for securing data needed for instrument and control design.

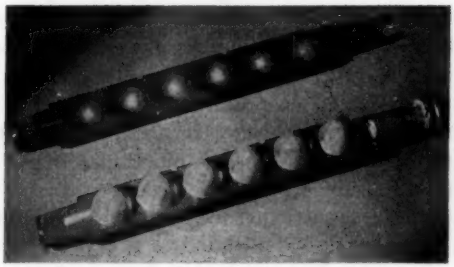


Preliminary scale model of Helicopter Flight Simulator Platform being developed and built for the Bell Helicopter Corporation, of Fort Worth, Texas, under the Army Navy Instrument Program.

Basic Research

The year 1957 was notable because of a public awakening to the importance of basic research. That same year saw continuation of several programs of basic research in The Franklin Institute Laboratories and the initiation of additional work of this type.

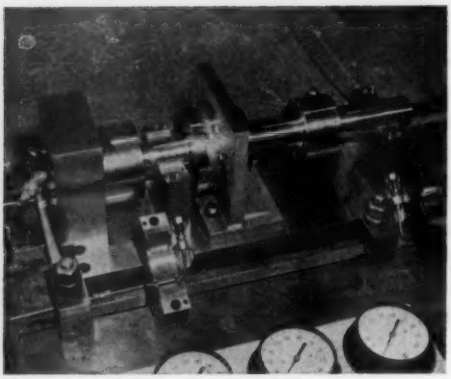
A large part of the continuing program in solid state physics is basic research: thermoelectric effects in solids; the properties of single-domain ferromagnetic particles; and the physical properties of extremely pure specimens of titanium and zirconium purified by zone melting. A new project was started on a study of the plastic deformation of metals by the observation of single dislocations. These studies have yielded new information that has been or is about to be published.



Spherical single crystals used for metallurgical research.

Some of these investigations of solid state phenomena are close to "pure" research—learning more about nature without regard to any possible application of the results. Others, such as the investigation of thermoelectric properties of alloys, are motivated by the hope for an eventual application.

Such motivation has prompted a comprehensive study of the phenomena involved in gas bearings. This project, sponsored jointly by eleven government agencies through the Office of Naval Research, has as its objective the accumulation of the basic engineering information about gas bearings, as the first step in the creation of a gas-bearing technology for use in modern aircraft and other machines.



Test rig for study of gas lubrication.

In mechanical engineering an exploratory investigation was made of the friction and wear characteristics of materials used for sliding seals at high speeds. Another project dealt with several aspects of the phenomena involved in friction between metal surfaces at very high rubbing speeds.

Basic research in rheology also leads to practical applications. New methods of measuring stresses in plastic materials that were developed during the past year will be of importance in the understanding and improvement of extrusion processes. Measurements of the dynamic properties of polyethylene showed that they are continuous through the melting range.

Univac Computing Center

The Univac Computing Center of The Franklin Institute was opened in January 1957. Its presence gives the Laboratories the opportunity to employ the latest techniques in the solution of involved engineering problems. It is also available to local industry for business computation.

Sharing Results of Research

The results of research in the Laboratories are made available to the public in a number of ways.

Publication of papers in scientific journals and trade publications gives the widest dissemination of the results. During 1957, 31 articles by members of the staff were published. Reprints of these papers are distributed gratis by the Laboratories.

Presentation of papers at meetings of technical societies also gives wide coverage. During 1957, 35 staff members presented papers at 23 local, national, and international technical meetings—in a number of cases by invitation.

The three symposia described on page 11 were planned by the Laboratories. In addition, four conferences were arranged for special groups to review research results. For instance, a classified symposium on electric initiators, held for Picatinny Arsenal on September 11 and 12, was attended by over 200 persons from 35 government agencies and 100 industrial companies.

Looking Ahead

The Franklin Institute has been performing research and development for more than a century and a quarter. These activities, however, were not made a formal department of the Institute until 1946. Since that time the Laboratories for Research and Development have occupied space in the main building of the Institute and in two other buildings.

Some of the space is not well-suited for laboratory operations. In general, serious overcrowding has occurred as the Laboratories have grown. Therefore, in the latter part of 1957, The Ballinger Company was engaged to make an architectural study of how the present and future space requirements of the Laboratories might be met.

This step has been taken because during the twelve years of their operation, the Laboratories have demonstrated their value to the government and to industry—as well as to the other programs of the Institute. The provision of improved facilities is expected to increase the efficiency of operation and also to give opportunity for further growth beyond that shown in the table on p. 246.

In planning this growth, mere increase in size of staff or numbers of projects is not the goal. Of more significance is an increase in the "centers

Growth of Laboratories

	1947	1956	**1957
Technical Personnel (Average)	215	242	259
Space Occupied, Dec. 31 (sq. ft.)	35,138	80,368	83,250
Government Sponsors	9	33	32
Government Contracts	22	109	109
Industrial Sponsors	18	109	93
Industrial Contracts	21	144	122
Gross Revenue:			
*Government Contracts	$1,404,000	$2,154,000	$2,690,000
Industrial Contracts	$ 33,000	$1,117,000	$ 960,000
Total	$1,437,000	$3,271,000	$3,650,000
Capital Expenditures	$ 220,000	$ 116,000	$ 125,000

* Including government sub-contracts from private industry.
** The financial figures for 1957 are preliminary ones, subject to audit adjustment.

of technical strength." Such a center comprises a specialist in a particular field, who furnishes the inspiration and leadership for a small staff doing basic research in that field. Around the periphery of this center are a number of project teams concerned with applied research and development related to the basic research. Members of the central staff assist the projects part-time; and in return, their own outlook is broadened.

The Laboratories today have centers of strength in a number of fields. The creation of additional ones will be possible as the Laboratories continue to grow.

BARTOL RESEARCH FOUNDATION

The Bartol Research Foundation is continuing its researches in nuclear physics, using large and small Van de Graaff generators, a proton spectrometer and a beta-ray spectrometer. The purpose continues to be analysis of nuclear structure in relation to energy levels and decay schemes. Plans for a new building to house the smaller generator more conveniently have been completed.

Linear accelerator and assembly used for electronic bombardment of surfaces for study of secondary emission.

The cosmic-ray program, active since 1927, is now associated with the International Geophysical Year. It is continuing to yield important results in connection with relations between cosmic radiation and solar activity. Bartol is operating a cosmic-ray station at Thule, Greenland, and has been allotted space for instruments in one of the proposed earth satellites. Neutron intensity is being measured aboard a ship at different parts of the world in order to obtain information on the primary cosmic radiation and the nature of the earth's magnetic field in outer space.

Researches are proceeding in connection with electrical conductivity induced by electronic bombardment and in connection with secondary electron emission from solid surfaces. A new two-million volt electron Van de Graaff generator, sponsored by the Office of Ordnance Research, is in an advanced stage toward completion.

BIOCHEMICAL RESEARCH FOUNDATION

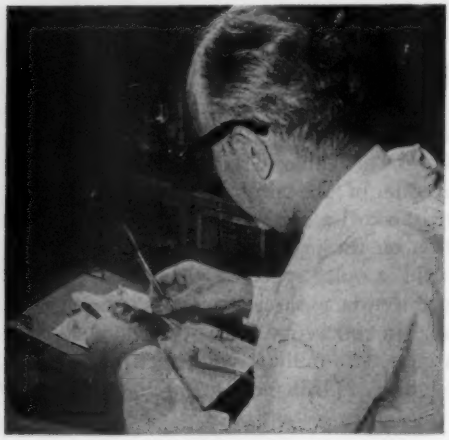
The Franklin Institute serves as trustee for the Biochemical Research Foundation located at Newark, Delaware; Dr. William G. Batt, Director.

The Biochemical Research Foundation is continuing the elusive task of tracking down fundamental information on the problem of cancer. A group of specialists in several disciplines has been conducting experimental research along several lines.

The enzyme-longevity-tumor investigations have shown that the enzyme activity alteration pattern, effected through diet, can be correlated with the incidence of spontaneous tumors and the ageing of the experimental animals. When the normal rate of change in enzyme activity was retarded, ageing was retarded; also delayed were the appearance of renal breakdown, pneumonia, possibly those conditions related to heart disease, and especially the formation of tumors.

The long range search for antimutagens has required the development of a practical screening procedure and the progress is most encouraging. Improvements in the design of apparatus for continuous growth brings the goal of *in vitro* culture of malignant cells a little closer.

Attempts have been made to develop tumor antigen-antibody tests for the establishment of the presence of antibodies. Tumor-host studies have been continued to determine why a rabbit tumor grows vigorously in the embryonic system of the chicken egg but is destroyed soon after the chick hatches. An attempt is being made to characterize the anti-tumor factor in the serum of very young chicks and to ascertain whether this factor is similar to that elicited in mature chickens by the injection of rabbit tumor cells. This factor will then be tested to determine whether it is tumor specific.



A pathological examination made in the course of the diet-enzyme-longevity-tumor program.

Organization

The preceding three sections of this report have reviewed the Institute's programs for 1957; this section will deal with the people who put those programs into effect and the means whereby it was done.

MEMBERS

The members of the Institute elect its Board of Managers and through their dues provide some financial support for the educational program. During 1957, the members adopted new By-Laws effective July 1957, giving all adult members the same privileges, and placing JOURNAL subscriptions for members on a voluntary, reduced-rate basis.

During 1957, 979 new members were added, bringing the total membership to 7430. Of these, 215 are Silver Members and 15 are Gold Members, having 25 and 50 years, respectively, of continuous membership in the Institute. Both groups of these "special" members were honored at dinners preceding Stated Meetings during the year.

Miss Darthela Clark, new "Gold Member," cuts anniversary cake at the November Stated Meeting Dinner. Watching are Dr. S. Wyman Rolph, Institute President, and Mr. Morton Gibbons-Neff, Chairman of the Membership Committee.

STAFF

The ultimate responsibility for carrying out the Institute's program falls on the staff. On December 31, 1957 there were 584 employees, representing a 5 per cent increase over the number of employees on December 31, 1956. During 1957, 6 entered military service, 2 died and 4 retired under the pension plan. Service pins were awarded to 47 employees: 1 for 30 years of service, 5 for 25 years, 1 for 20 years, 3 for 15 years, and 37 for 10 years. In accordance with the Institute's policy of helping employees to further their education, 58 employees received refunds totaling $4232 for college tuition fees.

A change in officers of the Institute occurred during the year. Mr. H. V. Bail, Treasurer since 1947 and Secretary since January 23, 1956, retired on April 30, 1957. He was succeeded by Mr. W. F. Jackson, Jr. as Secretary and by Mr. C. M. Waterbury as Treasurer.

Under the new By-Laws that became effective July 1, the Director of Laboratories was made an officer of the Institute. Dr. N. H. Smith was elected to this office.

FINANCES

In financing an operation as extensive and varied as that of The Franklin Institute, many sources of income must be sought. The Laboratories for Research and Development are self-supporting; but the educational and general activities are not. What income is received from these activities must be supplemented by donations of one kind or another. Such gifts are especially important today because of the emphasis being placed on scientific education for young people and the country's need for more pure and basic research.

The Institute has been fortunate during the past several years in being able to secure additional funds for its educational and general activities. The steady rise of operating costs, however, has caused a deficit each year, as shown by the accompanying table of income and expenses for these activities for the past three years:

Educational and General Activities

Expenses	*1957* *	*1956*	*1955*
Museum	$411,000	$357,286	$333,483
Library	105,000	94,114	82,546
Journal	42,000	40,708	33,226
250th Anniversary	13,000	24,981	—
General Expenses	42,000	58,600	75,537
Total	$613,000	$575,689	$524,792
Income			
Membership Dues	$ 68,000	$ 67,505	$ 64,276
Museum	193,000	171,649	155,315
Journal	33,000	26,258	21,351
Endowment Income	118,000	111,434	96,384
Appropriations from State and City	74,000	100,565	46,500
Friends of Franklin	52,000	45,495	39,440
Other Income	37,000	23,630	23,845
Total	$575,000	$546,536	$447,111
Deficit	$ 38,000	$ 29,153	$ 77,681

* The financial figures for 1957 are preliminary ones, subject to audit adjustment.

Although the increase in contributions is a gratifying indication of the general recognition of the value of the Institute's program, much more educational work needs to be done than the Institute can afford to do.

With the acquisition of the new Univac Computing Center in 1956, installation costs were $47,000. In 1957 the continuing operating costs during the build-up of full usage resulted in a deficit of $65,000. Operations for the Univac for November reached the break-even point for the first time and indications are that future operations will show greater improvement.

In addition to the gifts for operating expenses, the Institute received special gifts and additions to the endowment funds amounting to $12,000. Also, $19,675 were received for the construction of a paper industries exhibit.

The activities of the Bartol Research Foundation are financed in part by the income from the Bartol endowment, which income amounted to $128,000 in 1957. The balance of the funds needed to meet the total expenses of $325,000 was obtained from several government grants.

The financial statements of the Institute are audited each year by Messrs. Lybrand, Ross Brothers & Montgomery. The Auditors' reports when completed may be examined by members of the Institute in the office of the Treasurer.

PUBLIC RELATIONS

To keep the public informed of its activities, the Institute maintains a Public Relations Department for general activities and an Information Services Section within the Laboratories. The former prepares press releases on Institute affairs and arranges for special events; the latter acts as a clearing house for information about the Laboratories.

In November the Public Relations Society of America held its convention in Philadelphia and attended an open house at the Institute.

The members of the Institute's 250th Anniversary Committee were cited by the Philadelphia Chapter of the American Public Relations Association, for the outstanding work of that Committee. The Institute itself was cited by the Freedoms Foundation for its coordination of the 250th Anniversary Celebration. In May, "Benjamin Franklin's Unfinished Business," edited by Helen and Clarence Jordan (the chairman of the 250th Anniversary Committee), was published by the Institute as a permanent record of the world-wide participation in this unusual program, during the preceding year.

Gathered on the occasion of The Franklin Institute's contribution to the International Centenary of the birth of Nikola Tesla are, left to right, President S. Wyman Rolph, Executive Vice President John S. Burlew, Kenneth Swezey, and Samuel G. Hibben. The program, held at May's Stated Meeting, included: (1) a presentation by Mr. Swezey, a close personal friend of Tesla, of a copy of the collected letters, patents and articles of Tesla to the Institute; (2) a presentation of a resolution of the Institute's Board of Managers honoring the Nikola Tesla Museum in Yugoslavia, for sponsoring the Centenary, to Mr. Swezey for forwarding; and (3) Dr. Hibben's lecture, "From Tesla Until Tomorrow—The Magic of Making Light."

THE FRANKLIN INSTITUTE MUSEUM
A. C. CARLTON, DIRECTOR

"FROM ATOMS TO KILOWATTS"

BY

THOMAS COULSON

The most widely anticipated application of nuclear energy to peacetime purposes lies in the field of generating electric power. Several methods of using the tremendous energy released by nuclear fission have been developed and are in operation. To give the Museum visitor a clear conception how one of these methods operates, employing what is called a boiling water reactor, a special demonstration-lecture has been arranged with the co-operation of the Leeds & Northrup Company, of Philadelphia.

The primary requisite in the generation of electric power is heat energy to drive the turbine. Hitherto, this heat has been furnished by burning the fossil fuels, coal, gas, and oil. This heat is used to convert water to steam, which is then employed to drive the wheels of the turbine which turns the generator to produce electric power. The nuclear reactor will replace the fossil fuels as the source of heat.

The demonstration, called "From Atoms to Kilowatts," shows in a simulated reactor how the nuclear fission is controlled and held within manageable range. The visitor obtains a clear conception of the starting, control, and closing down of the reactor. The energy released by the atomic reaction is tremendous, but it can be regulated so that it is conveniently used to boil water surrounding the fuel. The steam is then used in the conventional manner. The demonstration shows how this is done.

Thus far, the demonstration is conducted by means of simulated devices, for it will be realized that the actual reactor, boiler, and turbine are too bulky to install in the Museum. After the flow of operations has been illustrated through these simulated devices, real instruments are employed as they could be used to control the operation of an actual nuclear power plant.

The growth of the number of active neutrons within the reactor begins in small quantities and increases to billions. This growth is simulated in the demonstration through an ingeniously designed electronic circuit which provides a response similar to that produced in the actual reactor. Three special instruments are furnished to measure this growth. Should the power level rise at a speed too fast for safety, the reactor is instantly and automatically shut down. The final stages of the operation are controlled automatically.

Additional instruments are provided for regulating the steam and feedwater flow to ensure that the turbine will run at 3600 revolutions per minute, at which speed the generator will produce the standard 60-cycle current. Other instruments measure the power plant's output through the regular electrical network.

An interesting feature if this complex instrumentation is that it can be controlled remotely from a central station which may be many miles away from the power plant and would control other power plants scattered over a wide area. The Museum demonstration illustrates this operation by remote control.

Apart from this demonstration the visitor will find conveniently located nearby several other exhibits associated with atomic physics. Here will be found a Van de Graaff generator which will, literally, make your hair stand on end; a Tesla coil, where the visitor can safely play with an electrical charge of 200,000 volts; and a Betatron, that wonderful instrument that can peer through twelve inches of armored plate to detect flaws in the metal.

THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT
NICHOL H. SMITH, DIRECTOR

SCIENTIFIC PROBLEMS SOLVED THROUGH UNIVAC

BY

THE STAFF OF THE ANALYSIS SECTION

The Univac Computing Center of The Franklin Institute comes under the Analysis Section of the Laboratories for Research and Development. The Center has been in operation approximately one year. During this time numerous scientific problems have been solved through the use of the Univac. Listed below are several of the early scientific applications successfully completed.

CALCULATION OF THE LOAD CARRYING CAPACITY OF A NOVEL DESIGN BEARING

This problem arose in the design of a hydrodynamically lubricated bearing of unusual shape. It was desired to compute the pressure distribution about the shaft (pressure *versus* angle) resulting from a variety of displacements of the shaft. The displacements varied in magnitude and direction so that the pressure was a function of the angle about the shaft, the magnitude of the displacement, and the angle made by the displacement. A side problem was to interpret the pressure distribution as a force distribution and to sum these to find a resultant force vector.

The final computation on UNIVAC took 4½ hours, while debugging and test runs took 25 minutes. The output, 242 tables of 37 entries each, took 25 minutes to print on the High Speed Printer. It has been estimated that the job would have taken more than one man year to complete by hand as opposed to four weeks elapsed time using UNIVAC.

AIRCRAFT PERFORMANCE CALCULATIONS

In order to evaluate the ground run and take-off performance of aircraft using jet-assist take-off units (JATO), two separate sets of aerodynamic equations were developed. Each set contained one dependent variable and five independent variables and involved powers; square roots; logarithms; trigonometric, exponential, and hyperbolic functions; and differentiation.

The complete performance calculation takes less than one hour of UNIVAC time for each aircraft at a cost of less than $1000.

NUCLEAR ENGINEERING CALCULATIONS

It was desired to find how the temperature of a slab varied from one face to the other while the temperature of one face was being raised proportional to time. The original differential equation was solved by a difference equation involving one dependent and two independent variables. Machine running time was about 25 minutes. The results of the computations were sets of 10 or 20 numbers representing slab temperatures from one face to the other where each set was for one instant of time. From the results, the temperature of any point in the slab at any time, after a starting time, could be read.

MOLECULAR WEIGHT CALCULATIONS

From time to time new methods of calculations bring about a more accurate table of atomic weights. It was desired to recalculate the molecular weights of about eight thousand chemical compounds using a revised atomic weight table.

The UNIVAC calculated:

1. The molecular weight of each compound
2. The percentage weight of each element in each compound
3. The percentage weight of any component compounds such as the anhydrous salt, water, etc.

The program facilitates an easy change when a new atomic weight table is published. A change to a revised atomic weight table would require approximately two man hours and five minutes of Univac time. The running time of the routine for the eight thousand compounds was 1 hour and 30 minutes.

SOLUTION OF LINEAR ALGEBRAIC EQUATIONS

The occasion arises at times in various research problems where it is necessary to solve linear algebraic equations. In many cases, the order of the set is less than 25.

In order to solve these equations, a UNIVAC routine was compiled entitled "Big Ben," which employs the Gauss elimination method.

UNIVAC computing time for the solution to an $n \times n$ system is:

$$\frac{.07n}{6}[2n^2 + 6n - 1] \text{ secs.}$$

DYNAMIC RESISTANCE STUDIES OF ELECTRIC INITIATORS

In a particular, simple electric circuit the six quantities: resistance, current, voltage, power, change in energy, and total energy dissipated, varied with time. Two differential equations were given describing the interrelationships of these quantities.

The equations were solved by a numerical approximation technique,

which was programmed, coded and debugged in 62 man hours. The Univac time to run the problem was 40 minutes.

STRESS-STRAIN CALCULATIONS

It was desired to calculate the stress on a given specimen from strain gauge readings at different points on its surface. These readings are mechanically recorded and must be corrected for the gauge factor of each particular gauge; the gauge factor setting on the recording instrument, and the scale factor setting on the recording instrument.

The calculations use constants such as Young's Modulus and Poisson's Ratio which must be changed each time the routine is used for a different material.

UNIVAC time for 1500 individual gauge readings is about 15 minutes. Previously these calculations were carried out on a desk calculator. The time required, without checking, was two man weeks.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

February 19, 1958

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Dr. John S. Burlew, Executive Vice President, called the meeting to order with approximately 280 members and guests in attendance, including members of the Instrument Society of America and a number of alumni of Phillips Academy, Andover.

Dr. Burlew stated that the minutes of the December 18, 1957 meeting were published in the January 1958 issue of the JOURNAL. There being no additions or corrections, the minutes were approved as published. Dr. Burlew announced that the minutes of the Annual Meeting of January 15, 1958 will be published in the February issue of the JOURNAL and will be presented for approval at the next meeting.

There being no further business, Dr. Burlew announced that this lecture was the second Charles S. Redding Lecture, established in honor of Mr. Redding, President of The Franklin Institute from 1941 to 1946 and now Chairman of the Board of the Leeds and Northrup Co.

Dr. Burlew then introduced the speaker of the evening, Mr. Kilshaw M. Irwin, Vice President in charge of Engineering at the Philadelphia Electric Company, and just named "Engineer of the Year." Dr. Burlew said that this was a fitting way for The Franklin Institute to participate in the "Engineers' Week" activities.

Mr. Irwin's talk, titled "More Power to Delaware Valley," included something of the growth of Delaware Valley requiring additional services of the Philadelphia Electric Company and how the electric company endeavored to keep abreast of the growth of the area. He spoke of the research and engineering work required in the planning and construction of the new Eddystone Station and illustrated with slides and extensive exhibit material various features of this new station. At the close of Mr. Irwin's talk guests were invited to examine the exhibit material on display on the floor of the Lecture Hall.

The meeting adjourned at 9:00 P.M. with a rising and enthusiastic vote of thanks to Mr. Irwin for his interesting presentation.

WILLIAM F. JACKSON, JR.
Secretary

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the doner. 525 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

HARRIS, CYRIL M., ED. Handbook of Noise Control. 1957.

LARGE, GEORGE ELWYN. Basic Reinforced Concrete Design; Elastic and Creep. 2nd Ed. 1957.

ASTRONOMY

KURTH, RUDOLF. Introduction to the Mechanics of Stellar Systems. 1957.

BACTERIOLOGY

BARKER, HORACE ALBERT. Bacterial Fermentation. 1956.

BIOGRAPHY

GRIFFITH, BEATRICE FOX. Pennsylvania Doctor. 1957.

BIOLOGICAL CHEMISTRY

PAULING, LINUS AND ITANO, HARVEY A. Molecular Structure and Biological Specificity. 1957.

BIOLOGY

HUTCHINSON, GEORGE EVELYN. A Treatise on Limnology. Vol. 1. 1957.

CHEMISTRY AND CHEMICAL TECHNOLOGY

BENEDICT, MANSON AND PIGFORD, THOMAS H. Nuclear Chemical Engineering. 1957.

CHU, JU-CHIN ET AL. Vapor-Liquid Equilibrium Data. 1956.

CLAPP, LEALLYN BURR. Chemistry of the Covalent Bond. 1957.

FAITH, WILLIAM LAWRENCE; KEYES, DONALD B. AND CLARK, RONALD L. Industrial Chemicals. Ed. 2. 1957.

FOX, SIDNEY W. AND FOSTER, JOSEPH F. Introduction to Protein Chemistry. 1957.

GREAT BRITAIN. NATIONAL COAL BOARD. Chemical Engineering in the Coal Industry. 1956.

HATCH, LEWIS FREDERIC. Higher Oxo Alcohols. 1957.

HOLNESS, H. Advanced Inorganic Qualitative Analysis by Semi Micro Methods. 1957.

KAPOOR, RAJENDRA M. AND MARTIN, JOSEPH J. Thermodynamic Properties of Chlorine. 1957.

MINRATH, WILLIAM R., ED. Van Nostrand's Practical Formulary. 1957.

REDDISH, GEORGE FULTZ. Antiseptics, Disinfectants, Fungicides and Chemical and Physical Sterilization. Ed. 2. 1957.

RENFREW, ARCHIBALD AND MORGAN, PHILLIP, ED. Polythene; the Technology and Uses of Ethylene Polymers. 1957.

RESEARCH. LONDON. Analysis; a Series of Papers Reprinted from *Research* Vol. 9, 1956. 1957.

SAGARIN, EDWARD, ED. Cosmetics, Science and Technology. 1957.

SWAN, GEORGE ALBERT AND FELTON, DESMOND GEOFFREY IVINS. Phenazines. 1957.

WALLING, CHEVES. Free Radicals in Solutions. 1957.

CRYSTALLOGRAPHY

FRIEDEL, J. Les Dislocations. 1956.

DOCUMENTATION

PERRY, JAMES WHITNEY AND KENT, ALLEN. Documentation and Information Retrieval; an Introduction to Basic Principles and Cost Analysis. 1957.

SHERA, JESSE HAUK; KENT, ALLEN AND PERRY, J. W. Information Systems in Documentation. 1957.

ELECTRICITY AND ELECTRICAL ENGINEERING

CANNING, RICHARD G. Installing Electronic Data Processing Systems. 1957.

COOKE-YARBOROUGH, EDMUND HARRY. An Introduction to Transistor Circuits. 1957.

DEWITT, DAVID AND ROSSOFF, ARTHUR L. Transistor Electronics. 1957.

GUILLEMIN, ERNST ADOLPH. Synthesis of Passive Networks; Theory and Methods Appropriate to the Realization and Approximation. 1957.

MAYERS, MORRIS A. AND CHIPP, R. D. Closed Circuit Television System Planning. 1957.

NEWTON, GEORGE C., JR.; GOULD, LEONARD A. AND KAISER, JAMES F. Analytical Design of Linear Feedback Controls. 1957.

SLURZBERG, MORRIS; OSTERHELD, WILLIAM AND VOEGTLIN, ELMO N. Essentials of Television. 1956.

SOMMERLATTE, FRITZ. Konstruktion von Schaltanlagen. 1956.

TURNER, RUFUS P. Transistor Circuits. 1957.

ENGINEERING

BRASUNAS, ANTON DE S. AND STANSBURY, E. E., ED. Symposium on Corrosion Fundamentals, a Series of Lectures. . . . 1956.

DREYER, GEORG. Formelsammlung zur Festigkeitlehre und Elastizitätslehre. Ed. 16. 1957.

FRANKEL, JACOB PORTER. Principles of the Properties of Materials. 1957.

GENERAL

SOUTHER, JAMES W. Technical Report Writing. 1957.

GEOLOGY

AMERICAN GEOLOGICAL INSTITUTE. Glossary of Geology and Related Sciences. 1957.

HISTORY

ROUSSEAU, PIERRE. Histoire des Techniques. 1956.

WILLOUGHBY, MALCOM FRANCIS. The U. S. Coast Guard in World War II. 1957.

MANUFACTURE

AMERICAN WELDING SOCIETY. Welding Handbook. 4th ed. Section 1. 1957.

CHAMBERLIN, N. S. AND OTHERS. Water Technology in the Pulp and Paper Industry. 1957.

KATZ, HYMAN H. Handbook of Layout and Dimensioning for Production. 1957.

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COHN, PAUL MORITZ. Lie Groups. 1957

FELLER, WILLIAM. An Introduction to Probability Theory and its Applications. Vol. 1. Ed. 2. 1957.

LUCE, ROBERT DUNCAN AND RAIFFA, HOWARD. Games and Decisions; Introduction and Critical Survey. 1957.

MIKHLIN, SOLOMON GRIGOREVICH. Integral Equations and Their Applications to Certain Problems in Mechanics, Mathematical Physics and Technology. 1957.

PEDOE, DANIEL. Circles. 1957.

SPRINGER, GEORGE. Introduction to Riemann Surfaces. 1957.

MECHANICAL ENGINEERING

FREESE, STANLEY. Windmills and Millwrighting. 1957.

GUNTHER, RAYMOND C. Refrigeration, Air Conditioning and Cold Storage: Principles and Applications. 1957.

METALLURGY

COMMITTEE ON METALLURGY. Engineering and Metallurgy. 1957.

METEOROLOGY

HALTINER, GEORGE J. AND MARTIN, FRANK L. Dynamical and Physical Meteorology. 1957.

NUCLEAR ENGINEERING

DUNNING, JOHN R. AND PRENTICE, BRUCE R. Advances in Nuclear Engineering. Vol. 1, Part 1 and Vol. 2. 1957.

PICKARD, JAMES K., ED. Nuclear Power Reactors. 1957.

OPTICS

CHU, CHIAO-MIN; CLARK, GEORGE C. AND CHURCHILL, STUART W. Tables of Angular Distribution Coefficients for Light Scattering by Spheres. 1957.

INTERNATIONAL CONFERENCE ON ELECTRON MICROSCOPY. Proceedings 3rd. 1956.

PHYSICS

BROGLIE, LOUIS, DE. Nouvelles Perspectives en Microphysique. 1956.

RIK, G. R. Einführung en die Massenspektroskopie. 1956.

U. S. NATIONAL BUREAU OF STANDARDS. Proceedings of the Cryogenic Engineering Conference. 1957.

VAN DER ZIEL, ALDERT. Solid State Physical Electronics. 1957.

WEBER, HAROLD CHRISTIAN AND MEISSNER, HERMAN P. Thermodynamics for Chemical Engineers. Ed. 2. 1957.

SCIENCE

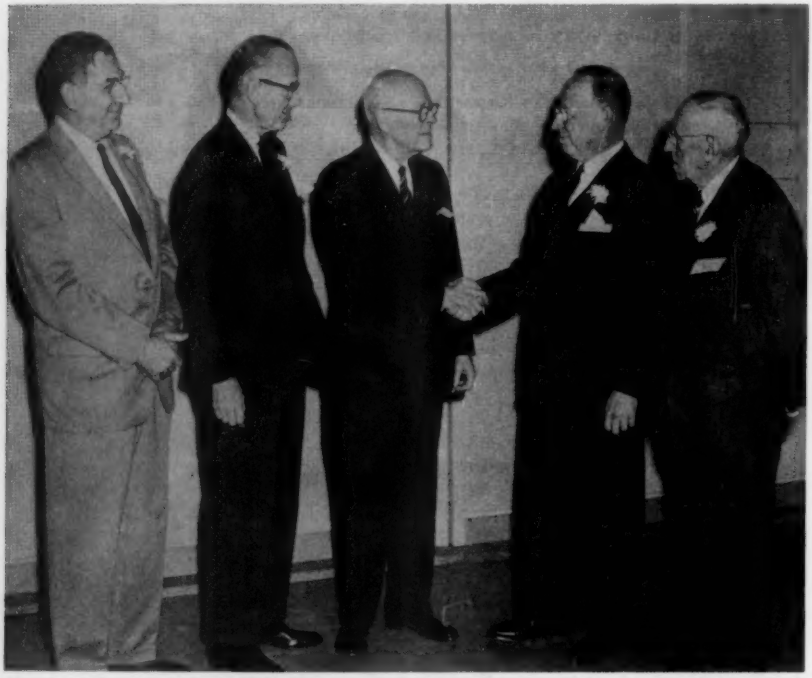
BONNER, FRANCIS T. AND PHILLIPS, MELBA. Principles of Physical Science. 1957.

MEMBERSHIP

On February 11, 1958, the Institute paid tribute to six men who had completed 25 years of consecutive membership during 1957. Four of the new Silver Members were able to attend the reception and dinner in their honor. As souveniers of the occasion, the new Silver Members received trays—gift of the Doehler-Jarvis Division of the National Lead Company—and initialled paper weights containing 1957 quarters.

Following dinner, Morton Gibbons-Neff, Chairman of the Membership Committee, introduced the four new Silver Members present: Arthur Fleischer; William A. Heine; W. Laurence LePage, newly elected President of the Institute; and E. Burke Wilford. He also recognized the following Silver Members of previous years who were present: James McGowan, Jr. (1909); J. D. Shattuck (1910); Howard S. Worrell (1912); Franklin P. McConnell (1913); Carl D. Pratt (1920); Morton Gibbons-Neff (1930); and Julius Neumueller (1931).

The dinner guests then heard Mr. Harold U. Landis, Jr., Field Engineer of the Ethyl Corporation, give a talk on "Fire Power," in which he explained the dangers of gasoline products and their use. This was one of the popular series of "Science is Fun" lectures for members only.



The Silver Members of 1957 who were present at the February 11 dinner, with the Chairman of the Membership Committee, Morton Gibbons-Neff. Left to right: Arthur Fleischer, W. Laurence LePage (newly elected President as well as a new Silver Member), Mr. Neff, E. Burke Wilford and William A. Heine.

BRIEF BIOGRAPHICAL NOTES ON THE 1957 SILVER MEMBERS

ARTHUR FLEISCHER—Senior Scientist in Electro-chemistry for Thomas A. Edison Industries Division of McGraw-Edison Electric Co. He holds about 20 patents.

WILLIAM A. HEINE—Clockmaker since 1918, he owns his own business in Philadelphia.

W. LAURENCE LEPAGE—Aeronautical Engineer, Management Consultant and President of The Franklin Institute. He designed the first licensed American autogiro and designed and built the first helicopters purchased by the U. S. Army.

ROBERT MCLEAN—President and Publisher of the *Philadelphia Evening and Sunday Bulletin*, and a Director of the Associated Press. He has received four honorary degrees and four awards, the most recent of which is the Edward Powell Award.

LEWIS G. SCHLEHNER—Steam and Electrical Engineer, who served the Institute as Building Superintendent for 20 years. He is the designer of the first completely motorized textile mill in the country.

E. BURKE WILFORD—President and Chief Engineer for the Pennsylvania Aircraft Syndicate Ltd. He received a Certificate of Merit from the Institute in 1939 and an honorary degree in 1947; he is Chairman of the Pioneers Convertible Aircraft.

NEW MEMBERS

The following elections were approved on February 6, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Philip Robert App
Andrew P. Asti
Harold W. Ballew
Roy A. Beers, Jr.
Ernest G. Beinhart
Donald G. Brown
Anthony Bruno
Alfred H. Buckalew
Carl J. Buehler
John B. Bullock
Charles R. Cabello
Samuel R. Caiazzo
James G. Calhoun
William R. Call
John J. Carroll
John Charambura
L. P. Clark
Sydney P. Clark
J. Rodman Clendenning
Philip W. Cloyes
H. E. Collyer
Wm. G. Crescenta, Jr.
W. E. Cromwell
D. Rogers Crosby
J. Alex Crothers
David Cullen
Elmer J. Cummings
Ellis I. Curley
Hector Daiutolo
George H. Dann
Ostap Danyluke
John B. Davis
Robert B. Day
Richard deCharms
Ralph W. Eberly
David E. Feinberg
Samuel Morse Felton, Jr.
G. Odell Fletcher
Edward A. Foehl
S. G. Holt
Margaret O. Hyde
Sam Jaffe
Anton Jurecic
Henry G. Keller
Friederich Gustave Krantz
Gustave H. Krauss
Walter Laforet
Daniel A. Logue, Jr.
Marion C. Mader
Allison Maggiolo
T. C. Manley
George K. Marple
C. E. McCarty
Ernest J. Mehr
Wm. W. Miller
James J. Moran
Harold C. Nagel
S. J. Niegowski
John T. Pinkston
M. J. Romanec
R. Barclay Scull
J. Brooks Semple
E. A. Shlauter
Decatur E. Shultz
Charles T. Smith
Nicol H. Smith
Randolph Ilgen Smith
Raymond L. Smith
Arthur Stander
Daniel D. Straff
James Vollmer
Robert G. Waite
Isadore Wiener
Robert P. Wolfson
Charles L. Wright
Anna R. Zappley
Tatiana E. Zell

EDUCATOR

Elizabeth F. deBerardinis
Robert M. McNair
Edward L. Rothstein
Frederic W. Van Name, Jr.

NON-RESIDENT

Alan Gresky

NECROLOGY

Caroline D. Bache '34
Everett P. Barnard '49
Joseph G. Denny, Jr. '45
William C. Hunnemann '46
William J. Latta, Jr. '38
Paul D. Merica '42
George L. Morehead '36
Willis R. Whitney '31

The following are Silver Members, having belonged continuously for twenty-five years or more. The dates are years of election to membership.

1909
Theobald F. Clark
Walton, Clark, Sr.
James McGowan, Jr.
T. Edward Ross

1910
Richard B. Brown
Joseph S. Hepburn
Joseph W. Lippincott
J. D. Shattuck
J. M. Weiss

1911
Seth B. Capp
W. H. Fulweiler
Charles Penrose

1912
James Barnes
Morris Llewellyn Cooke
Edwin Elliot
Walter Palmer
Julian S. Simsohn
Howard S. Worrell

1913
Edward Bartow
Sterling H. Bunnell
Irenee duPont
S. B. Eckert
Ralph E. Flanders
N. E. Funk
Lionel F. Levy
E. Mallinckrodt, Jr.
Frank M. Masters
Franklin P. McConnell
William Maul Measey
W. Chattin Wetherill
D. Robert Yarnall

1914
Russell L. Brinton
George S. Crampton
Ernest L. Huff
K. G. Mackenzie
George Wharton Pepper

1915
Thomas D. Cope
John C. Cornelius, Jr.
H. Jermain Creighton
Thomas W. Elkinton
Richard Howson
W. Wallace McKaig

1916
Henry B. Allen
James G. Detwiler
Zay Jeffries
Frank H. Sauer
B. E. Shackelford
Arthur Synnestvedt
Charles N. Weyl

1917
A. D. Chambers
Francis J. Chesterman
Walter S. Crowell
William J. Fitzmaurice, Jr.
Rolfe E. Glover, Jr.
Harold Goodwin, Jr.
Arthur W. Lowe
Frank S. MacGregor
Francis F. Milne, Jr.
C. H. Quinn
Sylvan D. Rolle
Robert L. Wood
Henry Woodhouse

1918
Charles H. Colvin
Walter O. Snelling

1919
Charles E. Brinley
Henry Colvin, 2nd
E. A. Eckhardt
George W. Furness
Frank H. Griffin
Hiram S. Lukens
J. Howard Pew
Edmund G. Robinson
L. K. Sillcox
W. Leigh Smith
John Sealy Townsend

1920
Robert Cameron Colwell
Chester Lichtenberg
P. S. Lyon
Haviland H. Platt
Carl D. Pratt

1921
Roland L. Andreau
Leonard T. Beale
J. Ed. Brewer
Arthur L. Day
Frederic Palmer, Jr.
Howard Stoertz
James Stokley
Edward R. Weidlein
S. Weinberg

1922
C. Douglas Galloway
Joseph F. Greene
C. E. K. Mees

1923
Henry Clay Gibson
Howard S. Levy
William C. Melcher, Jr.
John F. Metten
Jessie A. Rodman
Richard G. Sagebeer
H. Birchard Taylor
Francis B. Vogdes
Joseph J. Vogdes

1924
Peter Abrams
H. Carl Albrecht
Clement Starr Brinton
Lee Davidheiser
William G. Ellis
T. R. Harrison
Valentine Hiergesell
Karl F. Oerlein
Edward B. Patterson
Nicholas G. Roosevelt
Alfred O. Tate
Walter C. Wagner
James Lloyd Weatherwax
Alexander Wilson, 3rd
William Zimmermann

1925
Jack G. Binswanger
Marion Eppley
John Graham Foley
J. V. Giesler
C. R. Kraus
J. Kenneth W. Macalpine
H. F. Porter
W. F. G. Swann

1926
Charles B. Bazzoni
Neils Bohr
O. E. Buckley
William Findlay Downs
Henry C. Evans
Edward L. Forstall
Lewis H. Hendrixson
Charles S. Leopold
Alfred L. Loomis
J. H. Manning
Clement B. Newbold
O. M. Patton
Charles S. Redding
Dexter N. Shaw
Sydney L. Wright
Charles S. Wurtz, Jr.

1927
William Adam
T. K. Cleveland
Milton W. Deisley
William DeKrafft
Hiram B. Ely
Leslie Griscom
Alfred Iddles
John Van Gasken Postles

1927 (cont.)
Philip Sporn
Charles A. Stanwick
Charles A. Young

1928
William R. Dohan
Walton Forstall, Jr.
C. W. Hansell
Walter A. MacNair
Fred P. Nabenhauer
Joseph N. Pew, Jr.
William Rogers, Jr.
Frank N. Speller

1929
N. E. Bonn
Bernard Davis
Rupen Eksergian
Edward C. Haines
Malcolm Lloyd Hayward
Linn Helander
Randall Morgan, III
Harold F. Pitcairn
Edmund Rowland
Lawrence Saunders
A. Kelvin Smith
I. Melville Stein
William Stericker
Harden Franklin Taylor
Jacques L. Vauclain
Albert J. Williams, Jr.
Arthur M. Wilson, Jr.
C. T. R. Wilson
Enost E. Witmer

1930
Mildred Allen
William M. Barret
Aldus Fogelsanger
John Gardiner, Jr.
Morton Gibbons-Neff
Norman R. Gibson
Carroll Hodge
Walter J. Lehman
R. P. Long
S. B. Morehouse
William C. Peterman
G. E. Wiltbank

1931
Charles E. Berger
Carl Breer
Abraham D. Caesar
Daniel B. Curll, Jr.
Edward Dawson
J. Bennett Hill
Frederick C. Holtz
Penrose R. Hoopes
William Melas
Parry H. Moon
Julius Neumueller
Laurence P. Sharples

1932
Arthur Fleischer
William A. Heine
W. Laurence LePage
Robert McLean
Lewis G. Schlehner
E. Burke Wilford

The following are Gold Card Members, having belonged continuously for fifty years or more:

Fred H. Colvin '88
G. H. Clamer '91
J. H. Granberry '95
Edward Woolman '95
E. A. Muller '99
James M. Caird '01
Kern Dodge '02
John P. B. Sinkler '02
Charles H. Howson '03
Frank S. Busser '04
C. Mahlon Kline '04
George R. Hall '05
Darthela Clark '07
Frank Shaw Clark '07
W. M. C. Kimber '08

BOOK REVIEWS

SIXTH SYMPOSIUM (INTERNATIONAL) ON COMBUSTION. 943 pages, illustrations, 6¾ × 10 in. New York, Reinhold Publishing Corp., 1957. Price, $28.00.

This book contains 125 papers and summaries of three round-table discussions presented at a symposium at Yale University in August 1957. In contrast with preceding symposia in the series, the Sixth Symposium had a "comprehensive or broad agenda" reflecting the general growth of effort and broadening of interest in the combustion field without special emphasis on selected areas. The truly international character of this symposium is accentuated by the source of contributed papers on a world-wide basis: 28 from England, 16 from Japan, 13 from the U.S.S.R., 5 from India, 4 from Belgium, 3 from Sweden, and one each from Scotland, Holland and Germany.

Thirty-seven papers deal with fundamental problems of the structure and propagation of laminar flames, as contrasted with the Fourth Symposium in which one half of the papers were devoted to this field. Fifty-nine additional papers cover a broad range of combustion phenomena including turbulent flames, high-speed reactions, flame stabilization in fast streams, instability in combustion chambers, burning of solid fuels, combustion of explosives and solid fuels, evaporation and combustion of droplets and sprays. Special experimental techniques and instrumentation are discussed in ten additional papers. The list of technical papers is concluded with twenty articles on applied combustion dealing with fire extinguishment, chemical synthesis, rocket propellants, reciprocating engines and gas turbines. Owing to the scope and complexity of the subjects covered in the book, this review will be confined mostly to salient editorial features, rather than specific technical contents.

Several invited papers provide broad perspectives on the current status and trends in various combustion topics. The central problem of the theory of laminar propagation is reviewed by T. von Kármán. Experimental techniques and outstanding results of internal laminar flame structure investigations are discussed by R. M. Fristrom. The papers on the combustion of solid fuels are previewed in a summary report by A. A. Orning. The current status of fuel droplet studies in relation to spray combustion are critically reviewed by S. Kumagai. Methods of measuring high temperatures in high-velocity exhaust jets are discussed by I. Warshawsky.

The three round-table discussions feature: (1) future research problems, (2) high-speed reactions and (3) flame stabilization in fast streams. The unsatisfactory status of current knowledge in almost every combustion area is indicated by the variety of research problems proposed for future investigation. In regard to high-speed reactions, emphasis is placed on investigating deviations from thermal equilibrium by methods of mass spectrography, chemical sampling and analysis, flash photolysis and shock tube techniques. Flame stabilization in the wake of bluff bodies immersed in fast premixed gas streams appears to have been reasonably clarified by basic studies of the role of recirculation and mixing zones, but much remains to be done for improved understanding of stabilization under the technologically important conditions involving the presence of partially vaporized liquid fuel mists.

Among the contributed papers there apears a definite trend toward applied topics concerned with information which engineers can employ in improving the design of combustion apparatus. These topics include flame-holders, atomization, mixing, droplet and spray combustion, and scaling considerations. It is quite noteworthy that several of the papers dwell at length on the necessity of relating or organizing the knowledge obtained in the combustion studies in such form that the engineer can utilize the information in the design of combustion chambers. "Ends and Means in Flame Theory" by D. B. Spalding exemplifies this trend.

The book format is the same as that of previous publications based on the Third, Fourth

and Fifth Symposia. The publishers have performed well in composing, printing and high-quality reproduction of graphs, Schlieren, shadow and direct photographs.

While the book reflects the recent rapid growth of new research data and theoretical developments, in the latter field "no revolutionary discovery or concept or large generalization" has appeared, as noted by W. T. Olsen in the introduction to the text. Nevertheless, this book is an indispensable addition to the available references and data sources in the literature on combustion.

E. MAYER and R. B. DILLAWAY
Rocketdyne

BEHAVIOUR OF METALS AT ELEVATED TEMPERATURES. 122 pages, diagrams, $5\frac{1}{2} \times 8\frac{1}{2}$ in. New York, Philosophical Library, 1957. Price, $6.00.

This is an elementary text on high temperature metallurgy consisting of four lectures delivered at the Institution of Metallurgists Refresher Course, Llandudno, England in 1956. It is obviously not intended to be a comprehensive coverage of the high temperature behavior of metals.

The first paper by N. P. Allen covers the effect of high temperatures on the engineering properties of metals in a general way. A broad coverage is condensed into such a short summary that the text becomes confusing in places. On page 6 the author states, "It is not quite correct to say that at low temperatures the behavior is independent of the strain, for if the stresses are high enough, continuous yield will take place even at the lowest temperatures." Neither the implication nor meaning of this sentence is clear. Again on page 13, Dr. Allen describes temperature—log time curves as very often being good straight lines, yet neither of the illustrations that he uses bears this out.

The effect of temperatures up to 450° C. is covered by G. E. Meikle in the second paper. The author covers materials used in airframe construction. Short sections are devoted to the properties of many different commercial alloys. This makes rather confusing reading to one not familiar with trade designations of the alloys.

The third paper is a rather general summary of non-ferrous materials at temperatures above 500° C. by Dr. L. B. Pfeil.

The last paper, written by W. E. Bardgett, covers high temperature steels. The following classifications are discussed: Carbon Steels, Low Alloys Steels, Intermediate alloy steels, 12% chromium type steels, and Austenitic steels.

The author presents a broad picture and considers the effect of constitution on the high temperature properties. In dealing more with principles than with specific compositions and properties he is able to convey to the reader a useful summary of the classifications listed above.

Very little non-British work was used as reference material.

R. L. SMITH
The Franklin Institute Laboratories

RHYTHMIC AND SYNTHETIC PROCESSES IN GROWTH, edited by Dorothea Rudinick. 217 pages, illustrations, $6 \times 9\frac{1}{4}$ in. Princeton, Princeton University Press, 1957. Price, $7.50.

In July of 1956, the Fifteenth Symposium of the Society for the Study of Growth and Development was held at Brown University. Like the symposium, this volume of 10 chapters or papers is divided into three sections as follows: (1) recent advances in tissue culture methods, (2) cyclic activity in living organisms, and (3) biochemical evolution.

In the first section, T. T. Puck discusses his methods for cultivating single mammalian cells. With methods not unlike those for the growth of single bacteria or the isolation and cultivation of unicellular organisms to yield mass clonal cultures of many cells, growth in the laboratory has been established with cells isolated from human skin, liver, appendix, kidney, spleen, conjunctiva, and bone marrow. As with unicellular organisms, radiation-produced mutants of human cells were created which exhibited new nutritional requirements not possessed by the unirradiated stock. Continuation of these fascinating and far-reaching experiments should inevitably yield much information and understanding in regard to cellular activity in man and other mammals in health and disease. R. Dulbecco considers problems concerned with multiplication of RNA-containing animal viruses—those extremely small particles of organic matter which can multiply and which possess genetic continuity. His hypotheses on the basic func-

tion of RNA- and DNA-viruses should stimulate continued experimental work. R. M. Klein presents recent knowledge on the cultivation, growth, and differentiation of plant tissues in vitro.

In the second section, D. M. Prescott discusses the relations between cell growth and cell division and their synchrony. With precision methods, including the use of the Cartesian diver balance whereby it is possible to weigh a single living cell with accuracy, he has provided some answers to growth-division relationships. It is noted that the rate of growth in the cell rises to its greatest point immediately following cytoplasmic division and that the single nucleus of an ameba is capable of supporting a defined maximal amount of cytoplasm and no more. After this, growth ends and there follows nuclear and cytoplasmic division of the cell. It would seem profitable to this reviewer if similar studies were made on the giant multinuclear ameba, *Chaos chaos* in which a hundred or so nuclei are all in the same precise mitotic stage of division before the cytoplasm divides into two or more daughter cells. C. S. Pittendrigh and V. G. Bruce in an exiting and highly informative chapter develop the view, with examples, that there is a temperature-independent built in "biological clock" for time measurement in all or nearly all organisms including even the unicellular forms. There are some clear demonstrations of chronometry in organisms such as the time-compensated sun navigation of birds and the sense of time in bees to mention a few. Some other examples are less striking but are believed to be there nevertheless. The authors describe an ingenious oscillator model to explain and interpret this phenomenon in organisms. In a somewhat similar vein, E. Bünning considers diurnal cycles of activity in plants. In the interesting experiments reported, it is noted that if enzyme poisons are applied to plants for several days the metabolic processes may be eliminated without eliminating the persistent rhythms such as leaf movements or spore discharge. Indeed, there is no shifting of the phases of these rhythms which means that the "biological clock" is still "ticking" away.

In the final section dealing with biochemical evolution, the authors present a stimulating account that is based on known fact. At the same time it is also speculative—seen with the mind's eye, and theoretical—the fruit of reflection. H. Gaffron presents enough factual elements to propose a model showing the evolution of photosynthesis and the origin of life. Discussed is the role of the porphyrins and the belief that these and the chlorophylls were around a long time before the appearance of the modern proteins; these photocatalysts are still with us. Developed in a clear manner is the fact that photosynthesis—"a very improbable reaction"—is composed of several semi-independent sets of reactions and that light may be useful at times but not absolutely necessary. Since the green color appears in seedlings of higher plants only under the influence of light and many algae develop their chlorophyll in complete darkness, it is explained that the higher plants appear to have lost one link in the enzymatic chain leading to chlorophyll. H. F. Blum has contributed a valuable and thought-provoking chapter on the origin of self-replicating systems in living organisms, and B. L. Strehler discusses energy transduction problems in photosynthesis. Strehler proposes a mechanism linking the photochemical and biochemical parts to the process. The concluding chapter is a summary by Harlow Shapley dealing with the possibility of photosynthesis and life on other planets. Very likely many readers, like this one, will wish Harvard's distinguished scientist had extended his section but what he has to say in the five pages in interesting enough.

This valuable book contains the most recent discoveries and the latest thought in certain fundamental areas of the biological sciences. It will be read with profit by all who are interested in obtaining a fuller understanding of living things.

Each chapter, with the exception of the short, summarizing one, has a fairly comprehensive bibliography. Besides containing an index to the present volume, there is a valuable cumulative author-index of articles published in all previous symposia beginning with the first one of 1939.

RALPH WICHTERMAN
Temple University

RADIATION EFFECTS IN SOLIDS, by G. J. Dienes and G. H. Vineyard. 226 pages, diagrams, 6 × 9 in. New York, Interscience Publishers, Inc., 1957. Price, $6.50.

This volume of 226 pages is divided into six major portions. The first chapter, which is

actually the "Introduction," gives a very brief survey of the contents, and defines many of the topics to be discussed in the subsequent text. Ionization and energy loss of charged particles in matter are dealt with in the second chapter, and the treatment moves quickly into the domain of interest to solid state physics, embracing such topics as displacements, displacement cascades, development of interstitials, thermal and displacement spikes, etc.

From the beginning, it is made clear that the effects of heavy charged particles and neutrons are of prime interest in the case of this particular book. The uses of ultraviolet and infrared as primary excitants are purposely not treated, and the effects of gamma rays are dealt with as a special case. Numerous experiments are reviewed which concern the production of defects in solids by radiation. The gross effects of radiation such as changes in resistivity, thermal conductivity, volume, etc. are considered. Certain features of coagulation of defects are discussed as well as annealing of defects by application of heat, giving rise, for example, to recombination of vacancies and interstitials.

This book is lucid in presentation and very current in its content. Numerous references to recent works are given. The contents are of interest to those working actively in the field, to student, and to physicist desiring a comprehensive view of the subject.

C. E. MANDEVILLE
Bartol Research Foundation

INTRODUCTION TO ELECTRICAL APPLIED PHYSICS, by N. F. Astbury. 241 pages, $5\frac{1}{2} \times 8\frac{3}{4}$ in. New York, Philosophical Library, 1957. Price, $10.00.

This is an excellent condensed treatise on essential principles for applications to electromagnetic theory. The mathematics vector approach is merged along with specific physical explanations. Chapters II and III pertain to the electric and magnetic fields and thence in Chapter III are merged the discussion of units with the M.K.S. system and Maxwell's equations. In the discussion of electrical circuits, Laplace's transform is briefly introduced. Some detail analysis of magnetic phenomena is discussed wherein a rather unique approach to magnetic circuits is introduced. It is interesting that the author introduces "rotational" hysteresis which may become of greater interest in the magnetic property of metals.

Two welcome chapters are devoted to electromechanical and acoustic systems and to a brief theory of electron devices. The last chapter is devoted to the more difficult subject of electromagnetic waves and radiation, when the subject of retarded potentials is introduced.

It is difficult to set up a balanced presentation of essential analysis methods, including a clarification of units in a brief treatise, useful to both engineers and physicists. The author is to be congratulated on accomplishing this phase.

RUPEN EKSERGIAN
Consultant

THE NUMERICAL SOLUTION OF TWO-POINT BOUNDARY PROBLEMS IN ORDINARY DIFFERENTIAL EQUATIONS, by L. Fox. 371 pages, $6 \times 9\frac{1}{4}$ in. New York, Oxford University Press, 1957. Price, $9.60.

Here, Dr. Fox describes his personal experiences in the numerical computation, a field in which much of his research has been done. In the preface he states "I have included most of the worthwhile methods known[1] to me, which are suitable for pencil-and-paper and desk machine computation." Many worked examples are included in the text and good, clear descriptions are given of the computer's decisions as the calculations proceed. By working through the solutions, a novice should gain much experience and an appreciation of the limitations of a specific method, of the devices for error control, and of possible modifications to a technique.

The main part of the book describes various ways of approximating differential equations of orders one to four, including problems of the eigenvalue type and methods of solving the resulting approximate equation. Hence, the related topics of the production and proper use of finite difference equations, the solution of simulataneous algebraic equations and the determination of the eigenvalues and eigenvectors of arbitrary matrices are discussed. The methods for solving the simultaneous equations are the relaxation technique and the factorization of the matrix of coefficients into the product of a lower triangular matrix and an upper triangular matrix—this latter method being easy to use when the elements

[1] "Known" meaning first-hand experience and not acquaintance.

of the matrix are around the leading diagonal as is the case for finite difference approximations to ordinary differential equations. For the eigenvalue problem, iterative and relaxation techniques are described and illustrated.

Both linear and nonlinear differential equations are considered, and, in the worked examples especially, attention is given to the proper numerical treatment of the various possible types of boundary conditions. One chapter describes the ways of re-writing the differential equation or of choosing a finite difference approximation to illustrate the fact that boundary-value methods can be applied so that solutions can be obtained to the equations, for which step-by-step methods are generally used. Later, the author reverses the process and shows how step-by-step methods can be applied to the solution of boundary-value problems. Both of these latter methods are interesting, but the advantages of using them in preference to the more orthodox schemes are not obvious.

The precision of the solutions with regard to both round-off and truncation error is treated. Throughout the truncation, error is reduced by the use of the "difference-correction" method in preference to the more usual technique of reducing the finite difference interval. In the "difference-correction" method, a solution is obtained to a set of finite difference equations, which are approximations to the given differential equations. This solution is then differenced—the differences being an indication of the size of the truncation error. A correction to the original solution is then obtained by solving the original set of equations with these truncation errors as new right hand sides. For hand computation, it would seem to be the choice of the computer as to which methods he finds easier to apply. As far as the reviewer knows, this "difference-correction" method has not been programmed for an automatic digital computer.

This book, as the author states, "is intended for the practical computer and for the student seeking a career in computation." There is little "higher" mathematics or "rigorous" argument. Stress is laid on the computer's[2] experience and here there is excellent guidance from a person who has experimented with a wide variety of techniques. The information given will also prove useful to the more experienced computers[2] as it includes many good hints for improving techniques, and also describes different methods of attack which are well worth considering.

[2] The human being.

MARY LISTER
The Pennsylvania State University

STRENGTH OF MATERIALS, by F. R. Shanley. 783 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1957. Price, $8.50.

New strength of materials books appear on the market every year, and by and large they follow the same pattern, with only minor variations. Strength of Materials is pretty much a standard course, as it is taught in most U. S. schools, and textbooks generally reflect that standardization. The usual presentation consists of a one-semester sequence in which a number of standard design formulas are derived and the student given practice in their application through a set of conveniently simple problems. The philosophy of such a course is completely practical and echoes the needs of the structural or machine designer whose starting point will be a schedule of working stresses. There is nothing particularly wrong with this attitude, but there are many engineering educators today who feel that engineering courses can be, and should be, made more scientific, more rigorous and more penetrating, yet still remain engineering.

Professor Shanley's excellent text typifies this new look in engineering education thinking, and departs from the customary pattern in several respects. First of all, it cannot possibly be used for a one-semester course because it is too long. To do the book justice, a good year is required. Second, the author has gone far beyond the mere derivation of formulas in his effort to develop in the student a clear understanding of the physical behavior of loaded materials and an appreciation of the logical and philosophical aspects of the physical and mathematical thinking involved. Third, he aims to teach the student how to predict the actual failure of a structure, not how to design it. Very little attention is given to working stresses and other tools of the designer.

Physically, the book is about 750 pages long, and is divided into three parts, all of approximately equal length. The first section is a very comprehensive treatment of the

fundamentals of force and deflection and of stress and strain. The second covers the stress and strain analysis of individual structural members. The last deals with the failure of structural elements, and includes discussions of buckling, combined stress, stress concentrations, fatigue, and joints and fittings.

Professor Shanley has tried to be as comprehensive and as penetrating as possible without departing from fundamentals or introducing material beyond the scope of the usual student's mathematical preparation. In my opinion he has done a marvelous job. The student who learns his subject from this book should be helped much farther along the road to physical insight than will one limited to a more conventional text. In addition, the illustrations and general makeup of the book are first-class, and are a credit to the publisher.

The only sad thing about this fine book is that probably few schools will adopt it. Any textbook which requires expansion of a one-semester course to two semesters is not likely to be favored by administrators already squeezed to the limit for time.

WILLIAM C. YAGER
The Franklin Institute Laboratories

NUCLEAR STRIPPING REACTIONS, by S. T. Butler in association with O. H. Hittmair. 130 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc.; Sydney (Australia), Horwitz Publications Inc.; 1957. Price, $8.75.

This book of 130 pages includes most aspects of "direct reactions," those reactions which involve passage from initial to final quantum state without formation of a compound state. Stripping, pick-up, surface reactions, etc. are treated in very considerable detail. The various corrections such as Coulomb and nuclear scattering of the incoming and outgoing particles, and compound nucleus formation, are dealt with in a quantitative manner. In early portions of the book, the energy and angular momentum requirements for direct interaction are carefully discussed, and many helpful diagrams are presented which clearly depict the mechanisms of the various reactions. Classical analogies are offered which are extremely useful to the experimentalist in visualizing the sequence of events. Although this book is of importance to nuclear physicists generally, it should be of particular value to graduate students in theoretical physics, because the mathematical approach to and general methods of computation of such quantities as cross sections, reduced widths, angular distributions, and angular correlations are given in clear outline. Practically all of the subject matter is presented with clarity.

C. E. MANDEVILLE
Bartol Research Foundation

SOURCE BOOK OF INDUSTRIAL SOLVENTS. VOLUME 2: HALOGENATED HYDROCARBONS, by Ibert Mellan. 267 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $7.00.

Ibert Mellan's earlier book "Industrial Solvents" received a favorable reception and is indeed a very useful book. The author states its purpose as follows, ". . . to gather together and organize for ready reference sufficient material . . . to enable the reader to find useful information between the covers of one volume, to save him the trouble of searching through the voluminous literature which has grown up around the subject."

Exactly why the author and publisher have abandoned this format and are now issuing the *Source Book of Industrial Solvents* as a four volume work is hard to understand. Volume 2 presents description of halogenated hydrocarbons which are very similar to those in "Industrial Solvents," but adds azeotrope and solubility data, plus an assortment of material from industrial literature and a small number of original literature references. To the specialized user these books may therefore be a useful supplement, but to others the advantages of a one volume work would still be evident.

M. M. LABES
The Franklin Institute Laboratories

CATALYSIS, VOLUME 5, edited by Paul H. Emmett. 542 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $15.00.

Volume V of this series on "Catalysis" completes the treatment of most of the work on catalytic hydrogeneration which was initiated in Volume III and continued in Volume IV.

Chapter I on the reactions of carbon monoxide was written by Milton Orchin and Irving Wender. The variety of syntheses de-

scribed involve, for example, the formation of acids, esters, aldehydes, alcohols, unsaturated hydrocarbons, and heterocycles. Mechanisms for the reactions are treated primarily on the basis of formation of a carbonium ion and formation of a metallo-organic complex where metal salts are used.

Milton Orchin and Irving Wender again collaborated and with H. W. Sternberg in Chapter 2 to present the research on the "Oxo" Reaction. While this is a misnomer developed from early concepts of the general formation of oxygenated compounds from mixtures of hydrogen and carbon monoxide with hydrocarbons, which later has been shown to produce primarily aldehydes, the short term has appealed to the workers in the field to become a part of the literature. Essentially research on this reaction was stimulated by the results of the low-pressure hydrocarbon synthesis of Fischer-Tropsch in which small quantities of oxygenated compounds were produced. Its commercial importance is attested by the number of large commercial manufacturers, such as Standard Oil of New Jersey, Standard of Indiana, Tennessee Eastman, Carbide Chemicals and Gulf Oil Corp.

The first catalyst used in this synthesis was the conventional Fischer-Tropsch cobalt catalyst containing cobalt: thoria: magnesia: kieselguhr in the proportions 100: 5: 8: 200. Later research has shown that the oxo synthesis is catalyzed by soluble cobalt carbonyls. The authors have given appreciable consideration to the geometric and electronic structure factors of the carbonyls in relation to catalysis as well as effects of reaction conditions. They have included detailed descriptions of two procedures for preparing cobalt hydrocarbonyl. Oxo synthesis has been directed toward the production of many classes of organic compounds.

Direct Catalytic Synthesis of Higher Alcohols from Carbon Monoxide and Hydrogen (Chapter 3) is reviewed by G. Natta, U. Colombo, and I. Pasquon. The authors felt that sufficient detailed research results were not available to present a unified picture of the problems in this catalytic synthesis. Since the reaction results in the formation of a variety of alcohols which are difficult to separate, it has attracted only sporadic industrial interest.

The catalysts for these reactions which have been studied most were ZnO-Cr_2O_3 types with alkaline promoters.

Hilton A. Smith presents a very comprehensive treatment of the Catalytic Hydrogeneration of Aromatic Compounds in Chapter 4 including some 450 references to original work. The author reviews the mechanisms used in the explanations of the reactions with the various types of aromatic compounds including some heterocycles and fused rings.

The most detailed section of this volume is Chapter 5 on Hydrogen Isotopes in the Study of Hydrogeneration and Exchange by T. I. Taylor. This technique has been used extensively by a number of research workers and from its results more concepts on the mechanisms of catalysis have been stimulated to show how complex the reactions are. As yet there are few instances where clarification is widely accepted.

The final chapter (6) by Joseph B. McKinley concerns The Hydrodesulfurization of Liquid Petroleum Fractions. Industrial interest and important in processing petroleum fractions to control and remove sulfur constituents have fostered considerable study on this subject. With such variations in types and quantities of constituents in the sources of supply along with changes in structure that occur in processing, it seems hopeless to develop any reliable mechanisms in a system. However, a number of practical techniques of catalytic processing have been developed which the author discusses primarily by classes of compounds. The author has thoughtfully included thermodynamic and equilibrium data on the principal sulfur compounds found in petroleum as well as tables on the catalysts used in processing gasolines and various stocks and distillates.

The editor has organized the subject matter with a minimum amount of overlapping and the subjects are well cross referenced with other volumes in the series as well as within this work. In general, presentations are clear and concise and supported by extensive bibliographies. Teachers, advanced students, theoretical, and experimental researchers in the field will find this volume of constant value and indispensable along with other volumes of the series.

R. S. DALTER
The Franklin Institute Laboratories

BOOK NOTES

HIGH-TALENT MANPOWER FOR SCIENCE AND INDUSTRY, by J. Douglas Brown and Frederick Harbison. 97 pages, 5½ × 8½ in. Princeton, Princeton University Press, 1958. Price, $3.00.

A timely contribution, this small book examines the role of the corporation, the university and the state in developing the kind of talent needed in science and industry. The first of two essays comprising the book discusses the problem in the light of American needs; the second treats the problem as it is in the newly industrializing countries.

The authors believe that the kind of talent needed must be developed "through the systematic improvement of a whole complex of cultural, educational, and industrial institutions and processes." The book is a stimulating discussion of their proposals for such development.

BASIC ANIMAL HUSBANDRY, by John M. Kays. 430 pages, illustrations, 6 × 9 in. Englewood, N. J., Prentice-Hall, Inc., 1958. Price, $7.00.

The author, who teaches in the Animal Industries Department of the University of Connecticut, offers this unusual book for the beginning student, the farmer and the layman who is interested in animals.

The book covers the science of developing the most useful animals and includes breeding, feeding and practical aspects of marketing. After an introductory section, the book is divided onto four main parts treating beef cattle, sheep, swine and horses.

The text, amply illustrated with well chosen photographs, should interest many people not actively engaged in animal husbandry, for it contains useful information on judging animals, on meats, and on scientific breeding. The final chapter on horses contains a wealth of information of interest to devotees of horse racing.

INDUSTRIAL ELECTRONICS HANDBOOK, by R. Kretzmann. 298 pages, illustrations 6 × 9 in. New York, Philosophical Library, 1957. Price, $12.00.

The principles and properties of various classes of electronic tubes are described in this revised up-to-date text, together with typical applications and circuits. A chapter is devoted to each of the main types of application, such as electronic relays, counting circuits, etc. Containing many practical examples, this work describes in considerable detail the operation of each, with fully designed circuits. This handbook has proven invaluable to technicians engaged in the supervision or maintenance of industrial equipment as well as to industrial engineers in the mechanical and electronics fields.

INDUSTRIAL ELECTRONICS CIRCUITS, by R. Kretzmann. 194 pages, illustrations and diagrams, 6 × 9 in. New York, Philosophical Library, 1957. Price, $10.00.

This sequal to the "Industrial Electronics Handbook," deals with the circuitry of industrial electronic apparatus, and includes almost 200 carefully chosen circuits. Functions of various circuit elements are described, and comprehensive information is supplied on the actual component values. Examples are given of photoelectric control devices, counting circuits for various purposes, stabilizing circuits, switching and control circuits, amplifiers and oscillators, rectifying circuits and motor controls. Numerous photographs and illustrations are used in showing the design of the apparatus. Not only will this text find applications to the manufacturing industry, but physicists, engineers and technicians will have ample use for this handy guide to the development and design of electronic apparatus.

CATALYSIS IN PRACTICE, edited by Charles H. Collier. 153 pages, diagrams, 5 × 7½ in. New York, Reinhold Publishing Corp., 1957. Price, $3.95.

The seven papers in this book were first presented at the fifth "Experience in Industry" symposium, held in April, 1957. Sponsored by the Philadelphia-Wilmington section of the AICE and the School of Chemical Engineering of the University of Pennsylvania, these symposia cover practical aspects of the field and are directed at the young practicing chemical engineer.

This new volume on catalysis is limited—by reason of the complexity of the field—to

present technology within the chemical and petroleum industries. It covers the selection of a suitable catalyst for a given process, commercial preparation of industrial catalysts, fixed bed catalyst systems, moving bed processes, the economics of catalyst use, operating problems in catalytic processing, and a look at future prospects.

The authors are all experienced in the field and their contributions are well worthwhile.

NUCLEAR METALLURGY (VOL. IV). A SYMPOSIUM ON URANIUM AND URANIUM DIOXIDE. 143 pages, illustrations, 8½ × 11 in. New York, AIME, 1957. Price: $5.00 to members; $7.00 to non-members (paper).

This, the fourth volume on "Nuclear Metallurgy" issued by the Institute of Metals Division of the AIME, contains the sixteen papers presented at the fall meeting held in Chicago on November 6, 1957. The papers cover melting, casting, forging, rolling, cold-working, and welding of uranium, as well as the preparation and fabrication of uranium dioxide for fuel element use.

ELEMENTS OF TAPE RECORDER CIRCUITS, by Herman Burstein and Henry C. Pollak. 223 pages, diagrams, 5½ × 8½ in. New York, Gernsback Library, Inc., 1957. Price: $5.00 (hard cover); $2.90 (paper).

Aimed at both audiophile and technician, this book of ten chapters covers design factors of tape recorders, their performance characteristics, equalization circuits, oscillators, record-level indicators, and the minimization of noise and hum. The reader is assumed to have an elementary knowledge of electronics and audio. The authors, both audio engineers, have written the book from the design viewpoint, with emphasis on circuitry. The result is a practical treatise on the tape recorders currently on the market.

INDETERMINATE STRUCTURAL ANALYSIS, by J. Sterling Kinney. 655 pages, diagrams, 6 × 9 in. Reading (Mass.), Addison-Wesley Publishing Co., Inc., 1957. Price, $9.50.

This new text is aimed at the senior student or first year graduate student as well as the practicing structural engineer and architect. The reader should have sound knowledge of determinate structural analysis before beginning this text.

Fourteen chapters cover such topics as stability and determinateness of structures, basic concepts, methods for computing deflection, method of least work, column analogy, moment distribution, analysis of frames with nonprismatic members, slope-deflection method, influence lines, elastic arches and model analysis of structures.

ATOMIC ENERGY IN MEDICINE, by K. E. Halnan. 157 pages, plates;
ATOMIC ENERGY IN AGRICULTURE, by William E. Dick. 150 pages, plates;
ECONOMICS OF ATOMIC ENERGY, by Mary S. Goldring. 179 pages.
5 × 8½ in. New York, Philosophical Library, 1957. Price, $6.00 each volume.

In this first volume, the author discusses atomic energy in medicine with the use of radioisotopes (the radioactive materials in nuclear reactors) which has revolutionized many aspects of medical research and diagnosis. Examined also are radioactive "ash" produced by nuclear reactor furnaces and various types of accelerators or "atom smashing" machines. Reactors for nuclear energy research and the production of plutonium for the manufacture of atomic bombs, increasingly used in medical therapy to combat cancers and other growths, is also discussed. Written for those not necessarily having a great knowledge of either physics of medicine, this text may be found useful as an introduction to these subjects.

This second volume covers the use of atomic energy in agriculture. Dealing with the destruction and control of pests and diseases, the author delves into the speed-up of plant breeding and atomic radiation and food preservation.

According to this volume on economics of atomic energy, electricity generated by nuclear power will in the future be cheaper than coal or oil.

These three volumes in the "Atoms for Peace" series are intended for the lay reader, to acquaint him with peaceful uses of atomic energy.

PUBLICATIONS RECEIVED

The following books have been received for review in the JOURNAL. Lack of space prevents more than this listing.

MECHANICS OF MACHINERY, by C. W. Ham, E. J. Crane and W. L. Rogers. Fourth edition, 509 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.50.

GEOGRAPHY IN THE 20TH CENTURY, edited by Griffith Taylor. Third edition, 674 pages, plates, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $10.00.

INTRODUCTION TO HEAT TRANSFER, by Aubrey I. Brown and Salvatore M. Marco. Third edition, 332 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $6.75.

GEOMETRY OF ENGINEERING DRAWING, by George J. Hood and Albert S. Palmerlee. Fourth edition, 347 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $5.75.

LONG, LONG AGO, by Mary Lou Clark. 37 pages, illustrations, 5 × 8 in. New York, Pageant Press, Inc., 1957. Price, $2.00.

AN INTRODUCTION TO THE STUDY OF STELLAR STRUCTURE, by S. Chandrasekhar. Unabridged and unaltered reproduction of the first edition. 509 pages, 5¼ × 8 in. New York, Dover Publications, Inc., 1957. Price, $2.75 (paper).

APPLIED OPTICS AND OPTICAL DESIGN, by A. E. Conrady. Unabridged and corrected edition of the first edition. 518 pages, diagrams, 5¼ × 8 in. New York, Dover Publications, Inc., 1957. Price, $2.75 (paper).

LES MÉTHODES NOUVELLES DE LA MÉCANIQUE CÉLESTE, by H. Poincaré. Unabridged and unaltered republication of the volumes originally published in 1892, 1893 and 1899. Vols. I, II and III, 382, 479 and 414 pages, respectively, diagrams, 5½ × 8 in. New York, Dover Publications, Inc., 1957. Price: $2.75 each volume; $7.35 per set (paper).

TO MEMBERS AND THOSE WHO KNOW THE FRANKLIN INSTITUTE

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CURRENT TOPICS

"Cold" Fusion of Hydrogen Nuclei. —An experiment in the fusion of hydrogen nuclei without the need to contain large masses of gas at high temperatures was described to a Harvard Physics Department Colloquium recently by Associate Professor Karl Strauch, Harvard physicist. The experiment showed that although it is possible to make "cold" hydrogen nuclei fuse under laboratory conditions, the process cannot be used as a practical source of power.

Dr. Alick Ashmore, Dr. Ralph Nordhagen, Professor Strauch and Mr. Bryan Townes were working together at the University of Liverpool, observing the way in which mu mesons can catalyze fusion under laboratory conditions in such a manner that there is no need for maintaining a reaction chamber at a temperature so high that the toughest metals vaporize. But the experiments confirmed that the short life of the mu meson and other properties of this method of producing fusion required putting in more energy than could be withdrawn from the reaction.

The investigation was begun as a result of the discovery (by a group of scientists under the leadership of Professor Luis Alvarez at the University of California) of peculiar tracks in a hydrogen bubble-chamber attached to the Bevatron. A bubble chamber is a vessel filled with liquid which shows the tracks of charged particles as bubbles. This bubble chamber was inside a strong magnetic field which causes charged particles to curve. The particular track observed could not be explained by any reaction or particle then known.

Analysis of the curvature of the track and the gaps it possessed showed that the initial part of the track was due to a mu meson (or "muon"). The muon is a fairly recent addition to the subnuclear family, with a mass about two hundred times as great as the electron's. The muon is unstable, breaking down into an electron and two small chargeless particles called neutrinos in two millionths of a second.

This particle had attached itself to the nucleus of a deuterium atom, a form of heavy hydrogen, which occurs in nature as one hydrogen atom in every 5000. The deuterium atom, with the muon still attached, then joined with an ordinary hydrogen nucleus to form a mesic hydrogen-deuterium molecule in which the muon replaces an ordinary electron.

This mesic molecule is inherently unstable, in that energy is released when the hydrogen and deuterium nuclei fuse together, forming a helium nucleus.

However, this fusion with its subsequent energy release is possible only when the hydrogen and deuterium come close together. This event is highly unlikely in an ordinary molecule in which the two nuclei are held at a distance of about a hundred-millionth of a centimeter by an electron. But when the electron is replaced by a muon, this distance is reduced by a factor of two hundred, the ratio between the muon and the electron mass. The greater closeness of the nuclei in the mesic hydrogen-deuterium molecule increases the probability of fusion enough so that it can occur within the two millionths of a second lifetime of the muon.

In the events observed by the California group, the muon was thrown out of the helium nucleus after the fusion so that it could take part in other reactions. During this whole process the muon was unaffected, acting much like a catalyst in a chemical reaction.

Most fusion reactions take place in large masses of extremely hot gas, the sun or a hydrogen bomb for example. But here was a fusion reaction taking place in a vessel of liquid hydrogen, only a few degrees above absolute zero. Since the main problem in controlling hydrogen power is containing the million-degree temperatures involved, a "cold" fusion reaction looked very appealing.

Unfortunately, as the experiments at Liverpool showed, there are difficulties which will keep this reaction from commercial use. To be a practical source of energy, any reaction must release more energy than that required to start it. The two factors which prevent the muon-catalyzed fusion reaction from reaching practicality are, first, the short lifetime of the muon, which limits the number of reactions it can start, and second the unfortunate fact that the helium atom very seldom ejects the muon when the reaction is finished, but usually sends out a gamma ray instead. This cuts down the number of muons available to start new reactions, and the reaction in the hydrogen damps down and dies away.

Since there are no other particles known which can produce this catalytic effect, it seems at this time as if the only way to get useful power from hydrogen will be to contain a volume of gas at temperatures high enough to force the hydrogen nuclei close enough to each other to fuse. This will probably have to be done with a magnetic field, since all metals vaporize at the million-degree temperatures involved. Research on the problem continues: it has been estimated that the heavy hydrogen in the seas contains enough energy to supply one thousand times the world's present power needs for the next million years.

Allis-Chalmers and RCA Form New Engineering Group to Build Stellarator for Thermonuclear Research.—A new and unusual combined engineering staff organization known as C Stellarator Associates has been established by the Allis-Chalmers Manufacturing Company and the Radio Corporation of America to design and build a facility at Princeton University for advanced research into controlled thermonuclear reactions.

The installation, known as the Model C Stellarator, is based on the original concept of Dr. Lyman Spitzer, Jr. Since 1951, an AEC-sponsored program has been headed by Dr. Spitzer at Princeton University, with the objective of exploring means to harness the energy of nuclear fusion for peaceful purposes. The Model C Stellarator, located at the University's James Forrestal Research Center in Princeton, is scheduled for completion in 1960.

In a joint statement accompanying today's announcement, R. S. Stevenson, President of Allis-Chalmers, and John L. Burns, President of RCA, commented:

"We believe that the successful generation of useful power from controlled thermonuclear reaction would be one of the most significant advances in history, and we are pleased to have a part in advancing this important work.

"The enterprise is itself a new one for industry. It combines the best engineering talents of two complementary industrial organizations with the scientific talent of a major university."

The joint announcement stated that the engineering and administrative staffs of C Stellarator Associates will comprise approximately fifty people from both corporations. Their task will be to design the Model C Stellarator and coordinate fabrication of its components.

Commenting on the joint project, Mr. Linde and Mr. Herold, selected to head the program, characterized the effort as an extremely ambitious one involving a number of unprecedented large-scale engineering problems.

"The device will comprise a huge gas discharge in which it is hoped to generate temperatures as high as many millions of degrees," they said. "This is expected to provide the basic physical data necessary to achieve the design of still more powerful devices directed toward the design of fusion-power reactors. Such reactors, if achievable, would be free of the dangerous radioactive wastes which result from uranium or plutonium fission.

"Since it is known that the sun and stars produce vast amounts of energy by a fusion-power generating process, the proposed earth-bound system may be thought of as a stellar generator, or stellarator."

Subminiature Electrolytics.—The new Cornell-Dubilier Type EC aluminum-foil electrolytic capacitors were specifically designed for transistor and "low-B" tube applications where compactness is an important design factor. The smallest case size is 0.187 in. in diameter and ½ in. long, the largest size only 0.375 in. × 1½ in.

Type EC capacitors are available in ratings from 3 to 75 volts d-c. working, and in capacitance from 1.0 to 250 mfd., depending on voltage ratings. Operating temperature range is −20 C. to 65 C.

Type EC capacitors are housed in special tubular ceramic cases. The capacitor sections are sealed with a special cast resin, making them moisture-tight and heat-resistant. Tinned copper leads are welded to the anode and cathode tabs to insure low-resistance contact. Leads will not pull loose.

Stable capacitance, low dissipation factor, and low DC leakage current, assure dependable performance under all normal operating conditions.

Microspectrophotometer with Ultraviolet Microscope.—Neutronics Research Company, Waltham, Mass., has developed the first commercially available Microspectrophotometer with Ultraviolet Microscope.

A single UV-Visible source supplies a double-beam monochromator adjustable from 2300A to 6000A. Via time-sharing the specimen in the microscope is irradiated in such a way that reliable pilot-channel automatic calibration can be maintained. The transmission through the photo-multiplier and amplifying system controls a recorder, linked to the monochromator, which records transmission or absorption for areas as small as 1 u^2 at the flip of a switch. Hence, the automatic recorder plots transmission or absorption as a function of wavelength.

Land Mine Exploding Device.—Development of a tank mounted, mine exploding, roller which will help to keep armored units on the move in any future conflict, despite the tank's most powerful enemy, the buried land mine, has been announced by the Department of the Army.

Capable of withstanding antitank mine explosions, the roller clears pressure activated mines before the tank tracks reach the danger point. This makes possible more efficient utilization of tanks by eliminating time con-

suming search for and removal of mines. The roller was developed by the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va.

Fabricated of high strength steel, the complete roller is made up of a series of wheels 4 ft. in diameter and 6 in. thick mounted on independent shafts of axles. The tank in combat pushes a six wheel section in front of each track. The weight of the roller is sufficient to explode any buried mines equipped with pressure activated fuses that may be in its path.

The outer circumference of the roller wheels incorporates a special design which eliminates blank spots in ground coverage. Individual roller wheels are articulated to assist in more even ground pressure and to help absorb the shock of exploding mines.

Designed for field installation on the current standard tanks without modification to the vehicle, the attachment does not hinder the fighting capabilities of tank weapons. Assembly and installation can be rapidly accomplished by field personnel. Installation requires a crane or other lifting device as support equipment for assembly. The entire unit may be jettisoned from inside the tank in emergencies.

Heat-to-Electricity Converter.—Direct conversion of heat energy into electrical energy has been achieved in a unique electronic device developed at the General Electric Research Laboratory. The new thermionic converter invented by Dr. Volney C. Wilson takes advantage of the simple fact that electrons can be "boiled out" of a hot metal surface and used to produce an electric current directly. Experimental converters already have changed more than 8 per cent of the applied heat energy into electric power.

"Dr. Wilson's work has opened the door to new understanding of how electricity can be generated directly from heat without using machines that have moving parts. The thermionic converter is strictly one of those results of research that should be judged for the knowledge it provides and not at this time as a commercial product," according to Dr. Guy Suits, General Electric vice president and director of research.

Science has been seeking simple ways to convert heat into electricity ever since the usefulness of electricity was first discovered, Dr. Suits explained.

"Dr. Wilson's new thermionic converter is an extremely important scientific contribution, but neither he nor we want to imply that solar-powered space ships or inexpensive atomic-power plants are just around the corner because of this invention," Dr. Suits said.

"It is only natural to try to relate every new basic scientific discovery to some futuristic gadget. However, real research just doesn't ordinarily work that way. We cannot predict when–or if–the remaining scientific and economic problems inherent in direct heat-to-electricity conversion will be solved in a commercially-acceptable manner. But we can point out the tremendous contribution which Dr. Wilson has made toward the solution of one of the age-old riddles of the electrical industry," according to the General Electric research director.

Thermionic converters combine several known scientific principles in a unique manner. Two electrodes within the tube-like device are maintained at high—but different—temperatures. New approaches to the design of the electrodes, the materials used, and the gas environment within the envelope have resulted in a more efficient flow of electrons than ever noted before.

In a simplified explanation of his invention, Dr. Wilson says that when heat boils electrons out of the metal surface it is anologous to lifting water to the top of a hill. "If we let the water flow down the hill, it can do work—run a water wheel, for instance—but only if we can provide the water with a smooth, uninterrupted path down the hill. The thermionic converter essentially smooths the path of electrons from a hot electrode to a cooler one, and removes barriers which in the past have absorbed the energy before it could do useful work in an electric circuit."

Most previous methods of converting heat directly into electricity —without rotating machinery—have been based on the thermocouple, a device long used by scientists for measurement and control functions. Thermocouples utilizing a junction of two different kinds of metal produce small electric currents when heated, but their efficiencies normally are well below one per cent.

One obvious difference between the thermionic converter and the thermocouple is that in Dr. Wilson's device the metals are separated by a gas at very low pressure. There is an electrical flow between the electrodes, but there is less flow of heat than through a metal. Thus the electrodes can be at different temperatures, and the efficiency is greatly increased. It is hoped that thermionic converters ultimately may be able to change more than 30 per cent of heat energy directly into electricity.

Dr. Suits said that General Electric is investigating several approaches to the problem of converting heat directly into electricity. He described the opportunities for using solid-state semiconductor devices—thermocouples employing semiconductors—as well as the thermionic method.

"For nearly every vacuum-electronic tube device or system, there is a similar semiconductor method of accomplishing a similar electrical result. The semiconductor device corresponding to the thermionic converter is a thermo-junction which, upon heating, will produce a flow of current and power in an external circuit. This same semiconductor junction may, by passing current through it, be used to produce heat when the current flows in one direction and to produce cold when the current flows in the opposite direction. The latter case is the 'electronic refrigerator' which is under continuing study," Dr. Suits said. He described some recent progress by General Electric scientists in these research areas and said the thermionic converter represents "one of several promising investigations."

General Electric officials emphasized that thermionic converters are "experimental laboratory devices only" and are not considered ready for production.

"It is a significant research step," according to Dr. J. Herbert Hollomon, manager of metallurgy and ceramics research for General Electric, "but we must increase the thermionic converter's efficiency and adaptability to a variety of conditions if it is to become a major new electric power producer."

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